Contractual Frictions and Global Sourcing*

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Abstract

We generalize the Antràs and Helpman (2004) model of the international organization of production in order to accommodate varying degrees of contractual frictions. In particular, we allow the degree of contractibility to vary across inputs and countries. A continuum of firms with heterogeneous productivities decide whether to integrate or outsource the production of intermediate inputs, and from which country to source them. Final-good producers and their suppliers make relationship-specific investments which are only partially contractible, both in an integrated firm and in an arm's-length relationship. We describe equilibria in which firms with different productivity levels choose different ownership structures and supplier locations, and then study the effects of changes in the quality of contractual institutions on the relative prevalence of these organizational forms. Better contracting institutions in the South raise the prevalence of offshoring, but may reduce the relative prevalence of FDI or foreign outsourcing. The impact on the composition of offshoring depends on whether the institutional improvement affects disproportionately the contractibility of a particular input. A key message of the paper is that improvements in the contractibility of inputs controlled by final-good producers have different effects than improvements in the contractibility of inputs controlled by suppliers.

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1 Introduction

Insights from neoclassical trade theory and new trade theory have improved our understanding of the structure of foreign trade and investment. Recent developments in the world economy have sparked, however, an increased interest in new theoretical approaches designed to better understand the evidence about firms that organize production on a global scale. These developments include the growing role of multinational corporations in the global economy,[1] their engagement in more complex integration strategies,[2] and the growing share of intermediate inputs in trade flows.[3]

Although traditional theories allow for trade in intermediate inputs and for the emergence of international production networks,[4] they cannot explain some newly observed phenomena.[5] First, while the traditional approaches assume that firms are (for the most part) symmetrically structured within industries, the data exhibit substantial within-industry heterogeneity, both in the size distribution of firms and in their participation in foreign trade.[6] Second, in developing global sourcing strategies firms decide on where to locate the production of different parts of their value chains and also on the extent of their control over these activities. Which activities should they locate in the home country and which should they offshore? If they choose to offshore, should they engage in foreign direct investment (FDI) and import intermediate inputs within their boundaries or should they outsource the production of intermediates to independent foreign suppliers? As is well known from the work of Coase (1937), Williamson (1975, 1985), and Grossman and Hart (1986), these questions cannot be answered in a complete-contracting framework of the type used in traditional theories of international trade.

In Antràs and Helpman (2004) we developed a simple two-country Ricardian model of international trade in order to address some of these issues. In our model, firms in the North develop differentiated products. Then they decide whether to integrate the production of intermediates or outsource them. In either case firms have to decide in which country to source these inputs, in the high-cost North or the low-cost South. Production entails relationship-specific investments by both the final-good producers (or product developers) and their suppliers, and we assumed that the nature of these investments does not enable the parties to specify them in an enforceable contract. As in the work of Grossman and Hart (1986), we envisioned a world in which incomplete contracting creates inefficiencies even when the production of intermediate inputs is carried out by integrated suppliers. The key difference between integration and outsourcing is that only the former gives the final-good producer property rights over the fruits of the relationship-specific investments.

[1] The gross product (value-added) of multinational firms is roughly 25 percent of world GDP (UNCTAD, 2000). Leaving out the value-added generated by parent firms, about 10 percent of world GDP is accounted for by foreign affiliates, and this ratio has been increasing over time.

[2] See UNCTAD (1998) and Feinberg and Keane (2003).

[3] See for instance Hummels, Ishii and Yi (2001) and Yeats (2001). Feenstra and Hanson (1996) estimate that the share of imported intermediates increased from 5.3% of total U.S. intermediate purchases in 1972 to 11.6% in 1990. Campa and Goldberg (1997) find similar evidence for Canada and the U.K., but not for Japan.

[4] See Helpman and Krugman (1985, Chapters 11-13) and Jones (2000).

[5] See Helpman (2006) for a review of the newly observed phenomena and theoretical attempts to explain them.

[6] See Bernard and Jensen (1999) or Bernard et al. (2003) for evidence on heterogeneity in the exporting decision, and Bernard, Redding and Schott (2005) for evidence on heterogeneity in the importing decision.

Our model focused on the choices between integration and outsourcing and between domestic sourcing and foreign sourcing. In particular, we described an equilibrium in which firms with different productivity levels choose among the four feasible organizational modes: domestic outsourcing, domestic integration, foreign outsourcing (and thus imports of intermediate inputs at arm's length), and foreign integration (and thus FDI and intrafirm imports of inputs). We then studied the effects of variations in country and industry characteristics on the relative prevalence of these organizational forms.

In this paper we generalize the Antràs and Helpman (2004) model to accommodate varying degrees of contractual frictions.[7] In particular, we adopt the formulation of partial contracting from Acemoglu, Antràs and Helpman (2006). Final-good producers and their suppliers undertake a continuum of relationship-specific activities aimed at producing an intermediate input used in the production of the final good. A fraction of these activities is ex-ante contractible while the rest cannot be verified by a court of law and therefore are noncontractible. Both parties are bound to perform their duties in the contractible activities, but they are free to choose how much they invest in the noncontractible activities. Moreover, a party can withhold its noncontractible services at the bargaining stage over the division of surplus if it is not satisfied with the outcome. Every party's expected payoff in the bargaining game determines its willingness to invest in the noncontractible activities. Suppliers of intermediate inputs do not expect to receive the full marginal return from their investment in noncontractible activities, and therefore tend to underinvest in these activities relative to a complete-contracting benchmark. The larger the fraction of noncontractible activities is, the larger the distortions in production are.

We allow the degree of contractibility to vary across inputs and countries.[8] As in Antràs and Helpman (2004), we describe equilibria in which firms with different productivity levels choose different ownership structures and supplier locations. We then study the effects of changes in the quality of contracting institutions on the relative prevalence of these organizational forms.

We begin the analysis with a closed economy in which an organizational choice boils down to outsourcing versus integration. We show that, as in our previous work, the relative importance of the inputs provided by different parties is a crucial determinant of the "make-or-buy" decision.[9] In particular, regardless of the degree of contractibility of the inputs, integration is profit-maximizing if and only if the production process is sufficiently intensive in the input provided by the final-good producer. The new interesting result is that the degree of contractibility of different inputs plays a central role in the integration decision. Improvements in the contractibility of an input provided by the final-good producer encourage outsourcing while improvements in the contractibility of

[7] Using data on the activities of U.S. multinational firms, Yeaple (2006) presents evidence supporting some salient cross-industry implications of our model. In particular, he finds that the share of intrafirm imports in total U.S. imports (a measure of the relative prevalence of FDI over foreign outsourcing) is higher in industries with high R&D intensity and high productivity dispersion. Although the generalized model developed in this paper also implies a positive correlation between the share of intrafirm trade and productivity dispersion, it implies a more nuanced correlation with R&D intensity.

[8] But we maintain the standard assumption that the set of available contracts does not vary with firm boundaries.

[9] See also Grossman and Hart (1986) and Antràs (2003, 2005).

an input provided by a supplier encourage integration. This contrasts with the transaction-costs literature (e.g., Williamson, 1975, 1985), where any type of contractual improvement tends to favor outsourcing.

We next extend the analysis to a two-country world in which final-good producers can contract with suppliers in their home country, North, or a foreign country, South. Wages are higher in North, but North has better contracting institutions in the sense that larger fractions of activities are contractible in North. Although final-good producers always locate in North and make their investments there, we allow the contractibility of these investments to be a function of the location of suppliers. This reflects the notion that certain clauses of a contract may be harder to enforce when the contract governs an international transaction or when one of the parties resides in a country with weaker contracting institutions.

Having constructed equilibria in which firms with different productivity levels sort into different organizational forms, we proceed to study the effects of improvements in contractibility on the relative prevalence of these organizational forms. We first derive the result that improvements in contractibility in South raise the share of Northern firms that offshore the production of intermediate inputs. In contrast, improvements in contractibility in North reduce the share of offshoring firms. These results are in line with recent arguments that the quality of contracting institutions impacts comparative advantage (see Helpman (2006) for a summary); the work of Nunn (2006) provides empirical support.[10]

We also show, however, that the effect that changes in contractibility have on the relative prevalence of particular organizational forms depends importantly on the nature of the contractual improvements. In particular, better contracting in South, which raises offshoring, may reduce the relative prevalence of FDI if the institutional improvement affects disproportionately the contractibility of inputs provided by the final-good producer. And better contractibility in South may reduce the share of firms engaged in offshore outsourcing when the contractual improvements are biased toward inputs provided by suppliers rather than the final-good producer. One has to be mindful of the impact that improvements in legal systems have on the contractibility of specific inputs when predicting the prevalence of particular organizational forms.

The rest of the paper is organized as follows. Section 2 develops our model of the firm in the presence of partial contracting. Section 3 studies the make-or-buy decision in a closed economy. Section 4 extends the analysis to a two-country world. Section 5 concludes.

2 Technology and Investment

In this section we generalize the model of the firm that we developed in Antràs and Helpman (2004) in order to accommodate varying degrees of contractual frictions. For this purpose we first focus on a single firm that produces a brand of a differentiated product, for which it faces a demand

[10] Nunn's (2006) estimates suggest that the impact of cross-country variation in contracting institutions on trade flows is of the same order of magnitude as the impact of cross-country variation in human capital.

function

$$q = Ap^{-1/(1-\alpha)}, \quad 0 < \alpha < 1,$$

where q is quantity, p is price, A measures the demand level, and α is a parameter that controls the demand elasticity; the larger α is the larger the elasticity of demand $1/(1-\alpha)$ is. As is well known, this form of demand results from constant elasticity-of-substitution preferences for brands of a differentiated product. This demand function yields revenue

$$R = A^{1-\alpha} q^{\alpha}. \tag{1}$$

Output q is produced with two inputs, headquarter services X_h and an intermediate input X_m, using a Cobb-Douglas production function

$$q = \theta \left(\frac{X_h}{\eta_h} \right)^{\eta_h} \left(\frac{X_m}{\eta_m} \right)^{\eta_m}, \quad 0 < \eta_h < 1, \quad \eta_m = 1 - \eta_h,$$

where θ represents productivity, which may vary across firms, and η_h is a parameter that measures the technology's headquarter intensity. As in Antràs and Helpman (2004), both inputs are brand-specific. That is, X_h and X_m have to be designed to precisely fit the needs of this brand; otherwise the services derived from the inputs equal zero. Moreover, an input designed to fit this brand cannot be usefully employed in the production of other brands of the product.

We follow Acemoglu, Antràs and Helpman (2006) in assuming that each one of the specialized inputs is produced with a set of activities indexed by points on the interval $[0,1]$, according to the Cobb-Douglas production function

$$X_j = \exp \left[\int_0^1 \log x_j(i)\, di \right], \quad j = h, m,$$

where $x_j(i)$ is the investment in activity i for input j. Investment in activities is input-specific: they can be used only to produce the input for which they were designed. We assume that activities connected with input j in the range $\left[0, \mu_j\right]$, $0 \leq \mu_j \leq 1$, $j = h, m$, are contractible, in the sense that the characteristics of these activities can be fully specified in advance in an *enforceable* ex-ante contract. The remaining activities $(\mu_j, 1]$ are not contractible.

The final-good producer has to supply headquarter services and she has to hire a supplier for the intermediate input. The supplier of X_m can be the firm's employee or an outside agent. At this point we put aside the question of whether the firm integrates the production of the intermediate input or outsources it; we will deal with this question later. For now note that in either case there is an agency problem, because *by assumption* the firm needs a supplier. The organizational form determines (i) fixed costs, to be specified later; (ii) variable costs of investment c_j per unit $x_j(i)$ for $j = h, m$ and $i \in [0,1]$, where c_h is borne by the final-good producer while c_m is borne by the supplier; (iii) the fractions of contractible activities μ_j, $j = h, m$; and (iv) the fraction $\beta_h \in (0,1)$ of the revenue that the final-good producer obtains at the bargaining stage, and the fraction $\beta_m = 1 - \beta_h$ of the revenue that the supplier of X_m obtains. We will discuss the details of alternative organizational forms in due course.

The timing of events is as follows:

1. The final-good producer enters the industry and finds out her productivity level θ.

2. The final-good producer chooses to leave the industry or stay and produce.

3. If she chooses to stay, the final-good producer chooses an organizational form.

4. The final-good producer commits to invest $\{x_{hc}(i)\}_{i=0}^{\mu_h}$ in the contractible activities of headquarter services and she offers potential suppliers a contract, which stipulates the supplier's required investment in the contractible activities of the intermediate input $\{x_{mc}(i)\}_{i=0}^{\mu_m}$ and an upfront payment of τ_m to the supplier, which can be positive or negative.

5. A large pool of potential suppliers can earn income w_m, and they are willing to accept the firm's contract if the payoff from supplying X_m is at least as large as w_m. This payoff consists of the upfront payment τ_m plus the fraction β_m of the revenue that they expect to receive at the bargaining stage, minus the cost of the inputs $\{x_m(i)\}_{i=0}^{1}$. Potential suppliers apply for the firm's contract and the firm chooses one supplier.

6. The supplier and the final-good producer simultaneously choose their investment levels $x_j(i) = x_{jc}(i)$ in the contractible activities $i \in [0, \mu_j]$, $j = h, m$, as specified in the contract, and both sides choose independently their remaining investment levels $x_{mj}(i)$, $i \in (\mu_m, 1]$, $j = h, m$, in the noncontractible activities.

7. Output
$$q = \theta \left(\frac{\exp\left[\int_0^1 \log x_h(i)\, di\right]}{\eta_h} \right)^{\eta_h} \left(\frac{\exp\left[\int_0^1 \log x_m(i)\, di\right]}{\eta_m} \right)^{\eta_m} \tag{2}$$
is sold and the resulting revenue is distributed between the final-good producer and the supplier in proportions β_h and β_m, respectively. (We will discuss the details of the bargaining later on.)

We seek to characterize a symmetric subgame perfect equilibrium (SSPE) of this 7-stage game.

To characterize an SSPE of this game first consider stage 6, in which the final-good producer and the supplier each choose their investment levels in the noncontractible activities. Using the revenue function (1), the final-good producer's problem is

$$\max_{\{x_h(i)\}_{i=\mu_h}^{1}} \beta_h A^{1-\alpha} q^{\alpha} - c_h \int_{\mu_h}^{1} x_h(i)\, di,$$

subject to equation (2), $x_j(i) = x_{jc}(i)$ for the contractible activities, and given investment levels $x_m(i)$ in the supplier's noncontractible activities. Similarly, the supplier's problem is

$$\max_{\{x_m(i)\}_{i=\mu_m}^{1}} \beta_m A^{1-\alpha} q^{\alpha} - c_m \int_{\mu_m}^{1} x_m(i)\, di,$$

subject to equation (2), $x_j(i) = x_{jc}(i)$ for the contractible activities, and given investment levels $x_h(i)$ in the firm's noncontractible activities. The Nash equilibrium of this noncooperative game yields

$$x_j(i) = x_{jn} \equiv \left(\frac{\beta_j \eta_j}{c_j}\right) \alpha R, \quad \text{for } i \in (\mu_j, 1],\ j = h, m, \tag{3}$$

for the noncontractible activities. It follows that the investment in noncontractible activities is

$$\begin{aligned} x_j(i)^{1-\alpha\omega} &= \alpha\theta^{\alpha} A^{1-\alpha} \eta_h^{-\alpha\eta_h} \eta_m^{-\alpha\eta_m} \left[\exp \sum_{\ell=h,m} \alpha\eta_\ell \int_0^{\mu_\ell} \log x_\ell(i)\, di\right] \\ &\times \left(\frac{\beta_j \eta_j}{c_j}\right)^{1-\alpha\omega_k} \left(\frac{\beta_k \eta_k}{c_k}\right)^{\alpha\omega_k}, \quad \text{for } i \in (\mu_j, 1], \quad j, k = h, m, \quad k \neq j, \end{aligned} \tag{4}$$

where $\omega_\ell = \eta_\ell (1 - \mu_\ell)$ for $\ell = h, m$, and $\omega = \sum_{\ell=h,m} \omega_\ell$. Note that ω_h measures the importance of the noncontractible activities of headquarter services in the production of the final good; it represents the elasticity of output with respect to x_{hn}. Similarly, ω_m measures the importance of the noncontractible activities of the intermediate input in the production of the final good; it represents the elasticity of output with respect to x_{mn}. These measures of the impact of the noncontractible activities on the production of the final good play an important role in our applications of the model. From the definition of ω_ℓ, noncontractible activities of input ℓ are more important the larger the weight η_ℓ of input ℓ is in the production function and the smaller the fraction of contractible activities μ_ℓ is. That is, ω_ℓ results from an interaction of technological features with contracting frictions.

For stage 5 of the game to generate a non-empty set of applicants for the supply of X_m the final-good producer needs to offer a contract that satisfies the suppliers' participation constraint, which is

$$\beta_m R - c_m \int_0^1 x_m(i)\, di + \tau_m \geq w_m, \tag{5}$$

where the left-hand side represents a supplier's payoff from forming a relationship with the final-good producer and the right-hand side represents his outside option before he forms this relationship. In this participation constraint the investment levels in the noncontractible activities satisfy equation (4); the investment levels in the contractible activities are $x_{jc}(i)$ for $i \in [0, \mu_j]$, $j = h, m$, as specified in the contract; and revenue R and output q are given by (1) and (2), respectively.

In stage 3 the final-good producer chooses the contract to maximize her payoff

$$\beta_h R - c_h \int_0^1 x_h(i)\, di - \tau_m,$$

subject to (1), (2), the participation constraint (5), and the incentive compatibility constraints (4). As long as there are no constraints on the upfront payment τ_m, the participation constraint is satisfied with equality at the solution to this problem. Therefore we can solve the upfront payment τ_m from the participation constraint treated as an equality and substitute the result into the final-good producer's objective function. Under these circumstances the final-good producer's choice of contractible investments is the solution to

$$\max_{\{x_h(i)\}_{i=0}^1, \{x_m(i)\}_{i=0}^1} \pi \equiv R - c_h \int_0^1 x_h(i)\, di - c_m \int_0^1 x_m(i)\, di - w_m,$$

subject to the incentive compatibility constraints (4) and the revenue and output equations (1) and (2). The solutions of $x_j(i)$ for $i \in [0, \mu_j]$, $j = h, m$, yield the contractible investment levels $x_{jc}(i)$ for $i \in [0, \mu_j]$, $j = h, m$. Using the first-order conditions of the maximization problem together with (3) they can be expressed as

$$x_j(i) = x_{jc} \equiv \frac{1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell}{1 - \alpha\omega} \left(\frac{\eta_j}{c_j}\right) \alpha R, \quad \text{for } i \in [0, \mu_j],\ j = h, m. \tag{6}$$

Comparing this equation with (3) we obtain:[11]

Lemma 1 *For every input $j = h, m$, investment in contractible activities is larger than investment in noncontractible activities, i.e., $x_{jc} > x_{jn}$, for $j = h, m$.*

Evidently, when investment in contractible activities exceeds investment in noncontractible activities the investment levels do not maximize overall profits, because the two types of investment are equally costly. Moreover, the relative investment levels in the contractible activities, $x_{hc}/x_{mc} = (\eta_h/c_h)/(\eta_m/c_m)$, are profit maximizing, while the relative investment levels in the noncontractible activities, $x_{hn}/x_{mn} = (\beta_h/\beta_m)(\eta_h/c_h)/(\eta_m/c_m)$, are not. The latter results from the fact that each party's return on its investment in noncontractible activities depends on its bargaining share β_j, and these shares are not necessarily equal. If they are equal, there is no distortion in the relative investment in noncontractible activities. Finally, note that the optimal investment levels for a profit-maximizing firm are $x_j(i) = (\eta_j/c_j)\alpha R$ for $j = h, m$. Therefore in the equilibrium the noncontractible activities are underinvested and the contractible activities are overinvested relative to the revenue level R.

This characterization of the contractible investment levels yields

$$x_{jc} = K_c \left(\frac{\eta_j}{c_j}\right)^{1 + \frac{\alpha\mu_j\eta_j}{1-\alpha}} \left(\frac{\eta_k}{c_k}\right)^{\frac{\alpha\mu_k\eta_k}{1-\alpha}}, \quad \text{for } j, k = h, m \text{ and } k \neq j, \tag{7}$$

where

$$K_c = \left(\frac{1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell}{1 - \alpha\omega}\right)^{\frac{1-\alpha\omega}{1-\alpha}} \left[\alpha\theta^\alpha A^{1-\alpha} \eta_h^{-\alpha\eta_h} \eta_m^{-\alpha\eta_m} \left(\frac{\beta_m \eta_m}{c_m}\right)^{\alpha\omega_m} \left(\frac{\beta_h \eta_h}{c_h}\right)^{\alpha\omega_h}\right]^{\frac{1}{1-\alpha}}.$$

[11] For a derivation of the profit function and proof of Lemma 1, see the Appendix.

This implies that the final-good producer's profits are

$$\pi = Z\Theta - w_m, \tag{8}$$

where $\Theta = \theta^{\alpha/(1-\alpha)}$ is an alternative measure of productivity, and

$$Z = (1-\alpha)\, A \left[\alpha^{\alpha} c_h^{-\alpha\eta_h} c_m^{-\alpha\eta_m} \frac{\beta_m^{\alpha\omega_m} \beta_h^{\alpha\omega_h} \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right)^{1-\alpha\omega}}{(1-\alpha\omega)^{1-\alpha\omega}} \right]^{\frac{1}{1-\alpha}} \tag{9}$$

is a derived parameter which is proportional to the demand level; it depends on the costs of inputs, on the bargaining shares, and on the importance of contractual frictions for headquarter services and intermediate inputs. As expected, the profits of the final-good producer are higher the higher the demand level A is, the lower the costs of inputs c_h and c_m are, and the less attractive the suppliers' outside option w_m is. In addition, her profits are lower the larger ω_h or ω_m is, which implies that her profits are higher the larger the fraction of contractible activities in headquarter services and/or in intermediate inputs is. These results are summarized in[12]

Proposition 1 *The profits of the final-good producer are decreasing in input costs c_j, $j = h, m$, declining in the outside option of suppliers, w_m, and increasing in the shares of contractible activities μ_j, $j = h, m$.*

Bearing in mind that $\beta_m = 1 - \beta_h$, note that profits are not monotonic in β_h, rather they are smallest when the revenue share of the final-good producer equals zero or one, and profits are higher for intermediate values. So consider the shares β_h and β_m that maximize profits. To find them, we maximize

$$\beta_m^{\alpha\omega_m} \beta_h^{\alpha\omega_h} \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right)^{1-\alpha\omega},$$

subject to the constraint $\beta_h = 1 - \beta_m$, and $\beta_h \in (0,1)$. The solution to this problem is unique; it is given by

$$\beta_j^* = \frac{\omega_j (1-\alpha\omega_k) - \sqrt{\omega_h \omega_m (1-\alpha\omega_h)(1-\alpha\omega_m)}}{\omega_j - \omega_k} \quad \text{for } j,k = h,m, \quad k \neq j; \tag{10}$$

and it implies that $(\beta_h^* - \beta_m^*)(\omega_h - \omega_m) \geq 0$, with strict inequality holding when $\omega_h \neq \omega_m$. That is, the final-good producer wants to give the supplier less than half the revenue if and only if the noncontractible activities in m are less important than the noncontractible activities in h. Moreover, β_j^* is increasing in ω_j and declining in ω_k, $k \neq j$, and $\beta_h^* = \beta_m^* = 1/2$ for $\omega_h = \omega_m$. In other words, the final-good producer wants to give the supplier lower shares of the revenue the less important noncontractible activities in m are and the more important noncontractible activities in

[12] For proofs of Propositions 1, 2, and 3, see the Appendix.

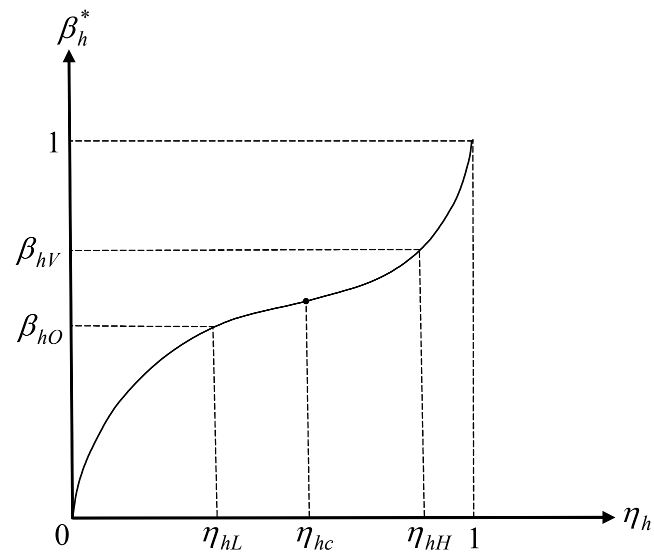

Figure 1: Bargaining shares and headquarter intensity

h are. Since $\omega_j = \left(1 - \mu_j\right) \eta_j$, it also implies that the final-good producer's optimal β_h^* is increasing in η_h, declining in μ_h, and increasing in μ_m. Finally, profits are rising with β_j for $0 < \beta_j < \beta_j^*$ and declining with β_j for $\beta_j^* < \beta_j < 1$. These results are summarized in

Proposition 2 *The optimal shares β_h^* and β_m^* have the following properties:*
(i) $(\beta_h^* - \beta_m^*)(\omega_h - \omega_m) \geq 0$, with strict inequality for $\omega_h \neq \omega_m$, and $\beta_h^* = \beta_m^* = 1/2$ for $\omega_h = \omega_m$.
(ii) β_h^* is increasing in η_h, declining in μ_h, and increasing in μ_m.
(iii) Profits are rising with β_j for $0 < \beta_j < \beta_j^*$ and declining with β_j for $\beta_j^* < \beta_j < 1$, $j = h, m$.

We will use these results in the following analysis.

3 The Make-or-Buy Decision

We now consider the tradeoff between integration and outsourcing; that is, whether to make the intermediate input in-house or outsource it to an outside supplier. In this section we focus on the case in which both choices are made in the same country, say the home country of the final-good producer. In this event input costs do not depend on the organizational form, nor do the degrees of contractual friction.[13] Moreover, the outside option of suppliers, w_m, does not depend on the make-or-buy decision. Under the circumstances we can focus on differences in the revenue shares β_j. In view of part (iii) of Proposition 2 the final-good producer prefers organizational forms with β_h closer to β_h^*.

[13] One could, of course, allow μ_m to vary with the internalization decision, but we prefer to avoid this complication.

Figure 1 depicts β_h^* as a function of η_h. In view of part (ii) of Proposition 2 this is an increasing function, and it is easy to verify that β_h^* approaches 0 when $\eta_h \to 0$ and β_h^* approaches 1 when $\eta_h \to 1$, as shown in the figure.

Now consider what determines the share β_h under outsourcing and integration. In stage 7 of the game the investment levels $x_h(i)$ and $x_m(i)$ are predetermined and therefore so are the input levels X_h and X_m of headquarter services and components. At this stage the supplier and the final-good producer bargain over the distribution of revenue R that they will receive when the final goods are sold in the market. Under outsourcing, X_m belongs to the supplier, while X_h belongs to the final-good producer. If the bargaining fails, output q equals zero and so does revenue. Moreover, given the high specificity of these inputs, which have no value outside the relationship, the outside option of every player equals zero. We assume that the parties engage in generalized Nash bargaining with a bargaining weight $\beta \in (0,1)$ for the final-good producer and $1-\beta \in (0,1)$ for the supplier. Therefore the solution to the bargaining game, which gives every player his/her outside option plus the bargaining weight times the ex-post gains from the relationship, delivers the final-good producer the payoff $0 + \beta(R - 0 - 0) = \beta R$. Namely, it gives her the fraction β of the revenue. By similar reasoning the supplier gets $(1-\beta)R$. It follows from this analysis that under outsourcing the final-good producer gets the fraction

$$\beta_{hO} = \beta$$

of the revenue, while the supplier gets the fraction $\beta_{mO} = 1 - \beta$.

Next consider integration. Under this arrangement the supplier is the final-good producer's employee and therefore the supplier does not own the intermediate input. As a result the outside option of the supplier equals zero. Following Grossman and Hart (1986), we assume that in the absence of the supplier's cooperation the final-good producer, who owns both X_h and X_m, cannot produce as efficiently with these inputs on her own as she can with the cooperation of the supplier. In particular, we assume that the final-good producer can produce on her own only a fraction $\delta \in (0,1)$ of the output that she can produce with the cooperation of the supplier, i.e., δq instead of q, where q is given in (2). In these circumstances the revenue is $\delta^\alpha R$ instead of R, where $R = A^{1-\alpha} q^\alpha$ is the revenue generated by q (see (1)). It follows that now the outside option of the final-good producer is not zero but $\delta^\alpha R$, and this outside option is smaller the larger is the efficiency loss from the departure of the supplier. As a result, the final-good producer's payoff from bargaining is $\delta^\alpha R + \beta(R - \delta^\alpha R - 0) = \beta_{hV} R$, where

$$\beta_{hV} = \beta + (1-\beta)\delta^\alpha$$

is the share of the revenue accruing to the final-good producer. The supplier obtains the revenue share $\beta_{mV} = 1 - \beta_{hV}$. Evidently, $\beta_{hV} > \beta = \beta_{hO}$, which means that the final-good producer gets a larger share of the revenue under integration than under outsourcing. In what follows, our analysis proceeds under the assumption that $\beta_{hV} > \beta_{hO}$.

Figure 1 depicts the revenue shares β_{hO} and β_{hV} and the headquarter intensities η_{hL} and η_{hH} for which each one of these shares maximizes profits. Part (iii) of Proposition 2 implies that all firms with intensity below η_{hL} prefer to outsource and all firms with intensity above η_{hH} prefer to integrate. By continuity, firms with intensity slightly above η_{hL} also prefer to outsource and firms with intensity slightly below η_{hH} also prefer to integrate. And we show in the Appendix that a unique critical intensity level exists between η_{hL} and η_{hH}, denoted in the figure by η_{hc}, at which a firm is just indifferent between outsourcing and integration. Firms with headquarter intensity below η_{hc} outsource and those with intensity above η_{hc} integrate.[14] This result is similar to our result in Antràs and Helpman (2004).

In order to study the impact of the quality of legal systems on industrial structure we need to understand how contractual frictions affect the make-or-buy decision. To this end first consider an improvement in contracting for intermediate inputs, reflected in an increase in μ_m, the fraction of contractible activities in the manufacturing of components. Part (ii) of Proposition 2 implies that this raises the optimal revenue share β_h. In Figure 1 this translates into an upward shift of the β_h^* curve. As a result, the critical intensity levels η_{hL} and η_{hH} decline. We show in the Appendix that the critical intensity level η_{hc} also declines.[15] The implication is that in response to improvements in contracting possibilities for components, more firms, i.e., firms with a larger range of headquarter intensities, choose to integrate. The reason is that with better contracting in intermediate inputs, final-good producers are less dependent on the power of the incentives they can offer suppliers, and for this reason outsourcing — which gives the suppliers stronger incentives than integration — becomes less attractive.

Importantly, the opposite happens when contracting improves in headquarter services. In this case part (ii) of Proposition 2 implies that the optimal revenue share β_h declines for every firm and the cutoff η_{hc} rises.[16] As a result firms with a larger range of headquarter intensities choose outsourcing over integration. The reason is that with better contracting in headquarter services it becomes more important to give suppliers better incentives, because contractual frictions now play a relatively more important role in components. In response more firms choose outsourcing, which gives the suppliers more powerful incentives.

These results are summarized in the following:

Proposition 3 *Let fixed and variable costs be the same under integration and outsourcing. Then: (i) There exists a unique headquarter-intensity cutoff* $\eta_{hc} \in (0, 1)$ *such that profits are higher under*

[14] The last statement follows from the fact that the critical value η_{hc} is unique, as we show in the Appendix. Note that at this critical value profits (8) are the same when the firm outsources or integrates, which means that the value of Z is the same under both organizational forms. Let $Z(\beta_h, \eta_h)$ be the value of Z when the final good producer receives a fraction β_h of the revenue and headquarter intensity is η_h. Then the definition of β_h^* implies that $Z(\beta_{hO}, \eta_{hL}) > Z(\beta_{hV}, \eta_{hL})$ and $Z(\beta_{hO}, \eta_{hH}) < Z(\beta_{hV}, \eta_{hH})$. For this reason the continuity of the function $Z(\cdot)$ implies that there exists a critical value $\eta_{hc} \in (\eta_{hL}, \eta_{hH})$ such that $Z(\beta_{hO}, \eta_{hc}) = Z(\beta_{hV}, \eta_{hc})$. The uniqueness of η_{hc} results from the fact that the ratio $Z(\beta_{hO}, \eta_h) / Z(\beta_{hV}, \eta_h)$ is declining in η_h.

[15] This stems from the fact that the ratio $Z(\beta_{hO}, \eta_h) / Z(\beta_{hV}, \eta_h)$ (defined in the previous footnote), which is declining in η_h at $\eta_h = \eta_{hc}$, also is declining in μ_m at $\eta_h = \eta_{hc}$.

[16] The last result stems from the fact that the ratio $Z(\beta_{hO}, \eta_h) / Z(\beta_{hV}, \eta_h)$, which is declining in η_h at $\eta_h = \eta_{hc}$, is increasing in μ_h at $\eta_h = \eta_{hc}$.

outsourcing for $\eta_h < \eta_{hc}$ and higher under integration for $\eta_h > \eta_{hc}$.
(ii) The cutoff η_{hc} is higher the larger μ_h is and the smaller μ_m is.

This proposition implies that whenever sectors differ by headquarter intensity and organizational choices do not affect fixed and variable costs, the make-or-buy decision does not depend on a firm's productivity, only on its sectoral affiliation. All firms in low headquarter intensity sectors choose outsourcing and all firms in high headquarter intensity sectors choose integration. Moreover, the fraction of sectors that choose integration is larger the larger is the fraction of contractible activities in components and the smaller is the fraction of contractible activities in headquarter services.

We next examine the impact of fixed costs on the make-or-buy decision of firms with different productivity levels. To this end suppose that there are different fixed costs of running an integrated or an outsourcing enterprise, which we denote by F_V and F_O, respectively. Under these circumstances we can replace the profit function (8) with

$$\pi_i = Z_i \Theta - w_m - F_i, \quad \text{for } i = O, V, \tag{11}$$

where i represents the organizational form, and Z_i is the derived parameter Z when evaluated at $\beta_h = \beta_{hi}$ and the industry's η_h. At this point we take variable costs c_h and c_m to be the same for both organizational forms, and therefore for a given industry, Z_i varies only with β_{hi}. Proposition 3 implies that $Z_O > Z_V$ for $\eta_h < \eta_{hc}$ and $Z_O < Z_V$ for $\eta_h > \eta_{hc}$.

Following Antràs and Helpman (2004), we now assume that integration involves higher fixed costs than outsourcing. Two opposing forces determine these costs. On the one hand managerial overload is larger in an integrated enterprise, because management has to pay attention to many more tasks. On the other hand there are economies of scope in management. If the managerial overload imposes larger costs than the costs saved due to economies of scope then $F_V > F_O$. In the opposite case $F_V < F_O$.

For concreteness we assume $F_V > F_O$. Under the circumstances profits from outsourcing are higher than profits from integration in all sectors with $\eta_h < \eta_{hc}$, independently of a firm's productivity level. The profit function π_O is depicted in Figure 2; it has an intercept at minus $w_m + F_O$ and a constant slope Z_O. The resulting profits are negative for $\Theta < \underline{\Theta}$. For this reason only firms with higher productivity manufacture in this industry. We also depict in this figure the profit function from integration, π_V; it has a lower intercept because $F_V > F_O$ and a lower slope because $Z_V < Z_O$. This shows that in industries with low headquarter intensity all profitable firms outsource.

Figure 3 depicts the two profit functions π_O and π_V in a sector with high headquarter intensity. Now the fixed costs still give outsourcing an advantage, as they did in the low headquarter intensity sector. But this is partly offset by lower-power incentives to the supplier, which helps the final-good producer (i.e., $Z_V > Z_O$). As a result, outsourcing dominates integration only for low-productivity firms, those with $\Theta < \Theta_O$.[17] It follows that firms with productivity below $\underline{\Theta}$ do not produce, those

[17] Note that in sectors with η_h close to η_{hc} the cutoff Θ_O is strictly above $\underline{\Theta}$, but in sectors with η_h close to 1 it

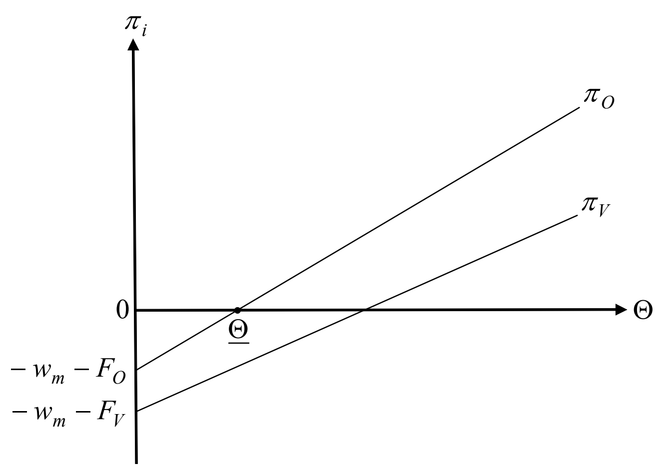

Figure 2: Profit function in a sector with $\eta_h < \eta_{hc}$

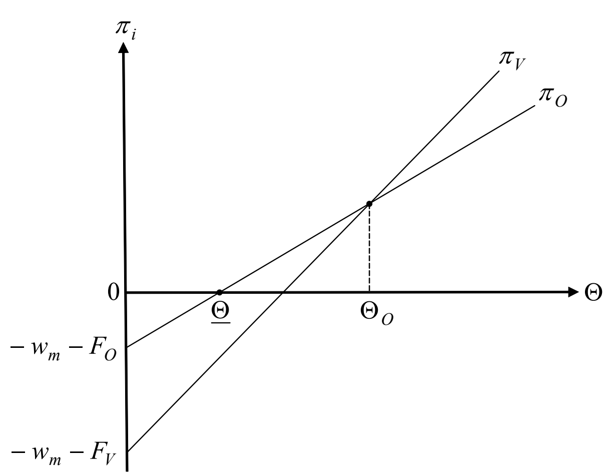

Figure 3: Profit function in a sector with $\eta_h > \eta_{hc}$

with productivity between $\underline{\Theta}$ and Θ_O outsource, and firms with productivity above Θ_O integrate.

Our results on the choice of organizational forms by firms with different productivity levels are summarized in

Proposition 4 *Let variable costs be the same under integration and outsourcing and let fixed costs be higher under integration. Then:*
(i) In every sector there exists a cutoff $\underline{\Theta}$ such that firms with productivity below $\underline{\Theta}$ do not produce.
(ii) In a sector with $\eta_h < \eta_{hc}$, all firms with productivity above $\underline{\Theta}$ outsource.
(iii) In a sector with $\eta_h > \eta_{hc}$, there exists a cutoff Θ_O such that all firms with productivity above this cutoff integrate. If this cutoff is above $\underline{\Theta}$, then all firms in the productivity range $(\underline{\Theta}, \Theta_O)$ outsource.

This proposition shows how fixed-cost differences between organizational forms interact with productivity differences across firms in shaping sectoral make-or-buy decisions. In economies with higher fixed costs of integration, high-productivity firms integrate and low-productivity firms outsource in sectors with high headquarter intensity. In sectors with low headquarter intensity all firms outsource.

Now consider the impact of contractual frictions on the relative prevalence of integration and outsourcing. Evidently, this analysis applies only to sectors with $\eta_h > \eta_{hc}$, in which the two organizational forms coexist. As in Antràs and Helpman (2004), we measure the prevalence of an organizational form by the fraction of firms that adopt it.

For this purpose let the cumulative distribution function of productivity be $G(\Theta)$. Then in sectors with $\Theta_O > \underline{\Theta}$ the fraction of firms that integrate is

$$\sigma_V = \frac{1 - G(\Theta_O)}{1 - G(\underline{\Theta})}.$$

Next suppose that Θ is distributed Pareto with shape parameter κ, so that[18]

$$G(\Theta) = 1 - \left(\frac{\Theta_{\min}}{\Theta}\right)^{\kappa} \quad \text{for } \Theta \geq \Theta_{\min} > 0 \text{ and } \kappa > 2. \tag{12}$$

Then

$$\sigma_V = \left(\frac{\underline{\Theta}}{\Theta_O}\right)^{\kappa}.$$

It follows that the share of integrating firms is larger the larger the ratio $\underline{\Theta}/\Theta_O$ is. From the definition of these cutoffs we find that

$$\underline{\Theta} = \frac{w_m + F_O}{Z_O},$$

can be below $\underline{\Theta}$. In the latter case all profitable firms in the industry integrate.

[18] There is a productivity distribution of θ, say $G_\theta(\cdot)$, and this distribution induces a distribution of $\Theta = \theta^{\alpha/(1-\alpha)}$, $G(\cdot)$. When θ is distributed Pareto with the shape parameter k then Θ is also distributed Pareto with the shape parameter $\kappa = k\alpha/(1-\alpha)$.

$$\Theta_O = \frac{F_V - F_O}{Z_V - Z_O}.$$

Therefore σ_V is larger the larger the ratio Z_V/Z_O is. We show in the Appendix that this ratio is decreasing in μ_h and increasing in μ_m. As a result, the share of outsourcing firms, which equals $1 - \sigma_V$, is increasing in μ_h and declining in μ_m. We therefore have

Proposition 5 *Let variable costs be the same under integration and outsourcing and let fixed costs be higher under integration. Then in sectors with $\eta_h > \eta_{hc}$ in which $\Theta_O > \underline{\Theta}$ the share of outsourcing firms is increasing in μ_h and declining in μ_m.*

It follows from this proposition that larger contractual frictions in headquarter services encourage integration and larger contractual frictions in components encourage outsourcing. For this reason overall improvements in the quality of the legal system, which raise the fraction of contractible activities in both headquarter services and components, may raise the relative prevalence of integration or outsourcing.[19] A key insight from this proposition is that contractual improvements per se do not bias the industrial structure toward outsourcing, because the differential impact of the improvement on contractual frictions in the two inputs plays an important role.

Note that Proposition 5 describes the impact of variations in contractual frictions on the prevalence of outsourcing even when there are general equilibrium effects, as long as the general equilibrium feedbacks do no impact the relative cost ratio $(w_m + F_O) / (F_V - F_O)$, because the unit costs c_h and c_m do not affect the Z_V/Z_O ratio, nor does the demand level A. It is therefore evident that this proposition holds in the general equilibrium of a one-factor economy, in which the fixed costs F_i, $i = O, V$, and w_m are proportional to the price of the factor. In fact, in this case we can think of w_m as the factor price.[20]

4 Foreign Sourcing

Next consider foreign sourcing. We assume that the final-good producer is located in North, which is a high-cost country. But North has good contracting institutions so the fraction of activities that are contractible is larger in North. Now a firm is not required to source the intermediate

[19] To clarify this point, let λ be an index of the quality of a country's legal system and let $\mu_i(\lambda)$, $i = O, V$, be increasing functions of this index. Then the marginal effects $\mu_i'(\lambda)$, i.e., the slopes of these functions, can differ substantially. We have no theory to tell how they differ, and it is clear from our analysis that there are differences that lead to a rise in the prevalence of outsourcing and other differences that lead to a rise in the prevalence of integration. Moreover, the shift in industrial structure may depend on sectoral characteristics, such as headquarter intensity. We show in the Appendix an example with $\mu_i(\lambda) = \lambda$ for $i = O, V$, in which the ratio Z_V/Z_O is rising in λ for $\eta_h = 0.4$ and declining in λ for $\eta_h = 0.5$, where both these η_h's are above η_{hc}.

[20] In our analysis we have assumed that $F_V > F_O$. Suppose instead that the fixed costs of outsourcing are higher than the fixed costs of integration. In this case we obtain the following results. First, in every sector there exists a cutoff $\underline{\Theta}$ such that firms with productivity below $\underline{\Theta}$ do not produce. Second, in a sector with $\eta_h > \eta_{hc}$, all firms with productivity above $\underline{\Theta}$ integrate. Third, in a sector with $\eta_h < \eta_{hc}$ there exists a cutoff Θ_V such that all firms with productivity above this cutoff outsource, and if $\Theta_V > \underline{\Theta}$, then all firms in the productivity range $(\underline{\Theta}, \Theta_V)$ integrate. Finally, in an industry in which some firms integrate and some firms outsource, we find that the share of outsourcing firms is increasing in μ_h and declining in μ_m, just as in Proposition 5. Hence, the effect of contractual frictions on the relative prevalence of integration or outsourcing is independent of the ranking of fixed costs.

input in its home country; it has a choice to source it in North or South. Unlike North, South is a low-cost country, but its contracting institutions are weaker and therefore smaller fractions of activities are contractible there. In what follows we denote with the superscript N variables that are affiliated with North and superscript S variables that are affiliated with South. Our assumption can therefore be represented by $\mu_j^N > \mu_j^S$ for $j = h, m$.

In addition we assume that the final-good producer has to produce headquarter services in North, but she can produce intermediate inputs in North or South, with $c_m^S < c_m^N$. In either case, i.e., independently of whether she produces components in North or South, she has the option to do so in-house or to outsource. When she chooses integration in South she engages in foreign direct investment (FDI). When she chooses outsourcing in South she engages in an arm's-length transaction. In the former case there is intrafirm imports of components; in the latter case there is arm's-length imports of components.

To simplify the analysis we assume that the revenue shares β_{hi}, $i = O, V$, are the same in North and South. As a result we can characterize the relative size of the cutoff η_{hc}, which now depends on whether components are produced in North or South. The cutoff η_{hc}^N is defined in the same way as before; it represents the headquarter intensity at which the final-good producer is indifferent between outsourcing and integration when the variable costs and fixed costs are the same in both cases, and the contractual frictions μ_j^N, $j = h, m$, are those prevailing in North. In other words, η_{hc}^N solves $Z_O^N = Z_V^N$, where in computing these Zs we use equation (9) evaluated at Northern variable costs. We have shown in the appendix that the ratio Z_O^N/Z_V^N does not depend on the variable costs and that it declines in η_h. As a result the solution to η_{hc}^N is unique.

We now define analogously η_{hc}^S as the headquarter intensity measure at which $Z_O^S = Z_V^S$, where Z_i^S represents the derived parameter Z in equation (9) evaluated at the unit cost of headquarter services in North, c_h^N, the unit cost of components in South, c_m^S, the Southern measure of contractual friction for headquarter services, μ_h^S, and the Southern measure of contractual frictions for components, μ_m^S. Now too the cutoff η_{hc}^S does not depend on unit costs and the ratio Z_O^S/Z_V^S is declining in η_h. As a result, the cutoff η_{hc}^S is unique. The implication is that in industries with $\eta_h < \eta_{hc}^S$ we have $Z_O^S > Z_V^S$, and in industries with $\eta_h > \eta_{hc}^S$ we have $Z_O^S < Z_V^S$. It then follows that among the firms who choose to offshore the production of intermediate inputs, those with $\eta_h < \eta_{hc}^S$ prefer to outsource, and those with $\eta_h > \eta_{hc}^S$ prefer to integrate, unless the fixed costs of integration and outsourcing are not the same.

Note that the ratio Z_O^S/Z_V^S differs from Z_O^N/Z_V^N only as a result of the difference between μ_j^S and μ_j^N for $j = h, m$. In the previous section (see Appendix for a formal proof), we have established that Z_O/Z_V is decreasing in μ_m and increasing in μ_h. As a result, the lower contractibility of components in South, $\mu_m^S < \mu_m^N$, tends to make the ratio Z_O^S/Z_V^S higher than the ratio Z_O^N/Z_V^N. On the other hand, our formulation implies that foreign sourcing also reduces the contractibility of headquarter services even though these are produced in North, $\mu_h^S < \mu_h^N$. The idea is that, as in Antràs (2005), all parts of a contract governing an international transaction are relatively harder to enforce. The lower contractibility of headquarter services associated with offshoring tends to make the ratio Z_O^S/Z_V^S lower than the ratio Z_O^N/Z_V^N. Overall, whether Z_O^S/Z_V^S is higher or lower than Z_O^N/Z_V^N depends on the relative magnitude of $\mu_h^N - \mu_h^S$ and $\mu_m^N - \mu_m^S$. Because it seems natural that the contractibility of an intermediate input is disproportionately affected by the contracting institutions of the country in which this input is produced, in the remainder of the paper we focus on situations in which the difference $\mu_h^N - \mu_h^S$ is low relative to the difference $\mu_m^N - \mu_m^S$. This allows us to establish the following result:

Proposition 6 *When $\mu_h^N - \mu_h^S$ is sufficiently smaller than $\mu_m^N - \mu_m^S$, the cutoff η_{hc} is higher when components are produced in South than when they are produced in North; that is, $\eta_{hc}^S > \eta_{hc}^N$.*

This proposition implies that when weak institutions in South have a stronger effect on the contractibility of components than headquarter services, then more sectors find outsourcing advantageous when they offshore than when they do not.[21] A direct corollary of this proposition is

Corollary 1 *When $\mu_h^N - \mu_h^S$ is sufficiently smaller than $\mu_m^N - \mu_m^S$, the slopes of the profit functions satisfy:*
(i) $Z_O^S > Z_V^S$ and $Z_O^N > Z_V^N$ for $\eta_h < \eta_{hc}^N$.
(ii) $Z_O^S > Z_V^S$ and $Z_O^N < Z_V^N$ for $\eta_{hc}^N < \eta_h < \eta_{hc}^S$.
(iii) $Z_O^S < Z_V^S$ and $Z_O^N < Z_V^N$ for $\eta_h > \eta_{hc}^S$.

We are now ready to characterize the joint offshoring and make-or-buy decisions. For this purpose we assume, as we did in the previous section, that the fixed costs of integration are higher than the fixed costs of outsourcing. Moreover, we assume that the fixed costs of offshoring are higher than the fixed costs of producing at home. In addition, we make the somewhat stronger assumption

$$F_V^S + w_m^S > F_O^S + w_m^S > F_V^N + w_m^N > F_O^N + w_m^N.$$

In this ordering the fixed costs of doing business in South are substantially higher than the fixed costs of doing business in North, and this difference overwhelms the South's cost advantage in w_m. The resulting profit functions are

$$\pi_i^\ell = Z_i^\ell \Theta - w_m^\ell - F_i^\ell, \quad i = O, V, \text{ and } \ell = N, S. \tag{13}$$

As in Antràs and Helpman (2004) it is now useful to study the equilibrium in sectors that differ by headquarter intensity, η_h.

4.1 Low Headquarter Intensity Sector

First consider an industry with headquarter intensity η_h smaller than η_{hc}^N. From Corollary 1 this implies $Z_O^\ell > Z_V^\ell$ for $\ell = N, S$. Given that the overall fixed costs of integration $w_m^\ell + F_V^\ell$ are

[21] Formally, the result follows from the fact that Z_O^ℓ/Z_V^ℓ is declining in η_h and μ_m^ℓ and increasing in μ_h^ℓ for $\ell = N, S$. Hence, by the implicit function theorem, η_{hc}^ℓ is increasing in μ_h^ℓ and decreasing in μ_m^ℓ. For $\mu_h^N - \mu_h^S$ sufficiently smaller than $\mu_m^N - \mu_m^S$, we thus have that $\eta_{hc}^N < \eta_{hc}^S$.

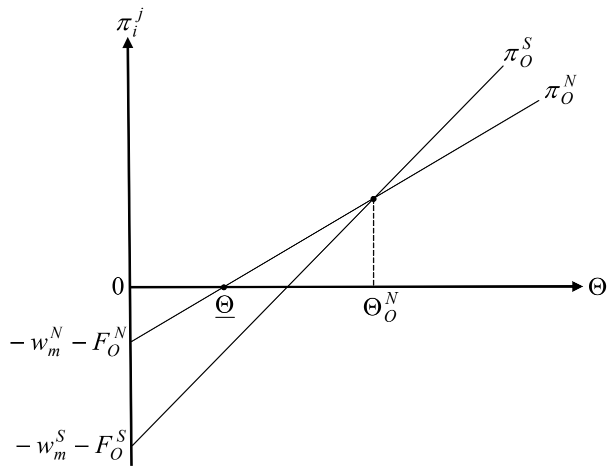

Figure 4: Profits from outsourcing when $\eta_h < \eta_{hc}^N$ and $Z_O^S > Z_O^N$

higher than the overall fixed costs of outsourcing $w_m^\ell + F_O^\ell$, it follows that in an industry with this headquarter intensity outsourcing dominates integration in North as well as in South, independently of a firm's productivity level, i.e., $\pi_O^\ell > \pi_V^\ell$ for $\ell = N, S$ and all Θ. Under the circumstances the effective choice is between outsourcing at home and outsourcing in South. Since the fixed costs of outsourcing in South are higher than the fixed costs of outsourcing in North, there is a tradeoff between these two organizational forms only if $Z_O^S > Z_O^N$; otherwise outsourcing in South dominates outsourcing in North. But the slope differential between the two profit functions from outsourcing is driven by two considerations. On the one hand the variable unit costs of producing components are lower in South, i.e., $c_m^N > c_m^S$, which raises Z_O^S relative to Z_O^N. On the other hand contractual frictions are higher in South, i.e., $\mu_j^N > \mu_j^S$ for $j = h, m$, which reduces Z_O^S relative to Z_O^N. In other words, the marginal profitability from higher productivity can be higher or lower in South depending on differences in unit costs and in contractual frictions.[22] In industries with $Z_O^S < Z_O^N$, all firms outsource in North. In industries with $Z_O^S > Z_O^N$, high-productivity firms outsource in South.

Figure 4 depicts the tradeoff for $Z_O^S > Z_O^N$. Firms with productivity below $\underline{\Theta}$ lose money either way, and they do not produce. Firms with productivity between $\underline{\Theta}$ and Θ_O^N outsource in North, and those with productivity above Θ_O^N outsource in South. This sorting pattern is similar to Antràs and Helpman (2004), except that now the case $Z_O^S < Z_O^N$ can also arise, in which all firms outsource in North. Note also that in the case depicted in the figure it is possible that all firms will outsource in South if $\underline{\Theta} > \Theta_O^N$; otherwise the two organizational forms coexist in the industry.

We now calculate the fraction of firms that outsource in South—that is, the fraction of firms

[22] In Antràs and Helpman (2004) we had no differences in contractual frictions, as a result of which we had $Z_O^S > Z_O^N$. There the assumption was $\mu_j^\ell = 0$ for $j = h, m$, and $\ell = N, S$.

that offshore—assuming the Pareto distribution of productivity (12). This fraction is given by

$$\sigma_O^S = \left(\frac{\underline{\Theta}}{\Theta_O^N}\right)^{\kappa},$$

where

$$\underline{\Theta} = \frac{w_m^N + F_O^N}{Z_O^N}, \tag{14}$$

and

$$\Theta_O^N = \frac{\left(F_O^S + w_m^S\right) - \left(F_O^N + w_m^N\right)}{Z_O^S - Z_O^N}. \tag{15}$$

It follows from these equations that σ_O^S is larger the larger the ratio Z_O^S/Z_O^N is. Naturally, this ratio is larger the larger the unit cost advantage of the South c_m^S/c_m^N is. Moreover, from Proposition 1, the ratio Z_O^S/Z_O^N is larger the larger is the fraction of contractible activities in South, either μ_m^S or μ_h^S, and the smaller is the fraction of contractible activities in North, either μ_m^N or μ_h^N. In summary, we have

Proposition 7 *Consider an industry with $\eta_h < \eta_{hc}^N$. Then no firm integrates and there exists a cutoff $\underline{\Theta}$ given by (14) such that firms with productivity $\Theta < \underline{\Theta}$ do not produce. In addition:*
(i) If $Z_O^S < Z_O^N$ then all firms with $\Theta > \underline{\Theta}$ outsource in North.
(ii) If $Z_O^S > Z_O^N$ then there exists a cutoff Θ_O^N given by (15) such that all firms with $\Theta > \Theta_O^N$ outsource in South, and if $\Theta_O^N > \underline{\Theta}$ then all firms with $\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ outsource in North.
(iii) If $\Theta_O^N > \underline{\Theta}$ then the fraction of offshoring firms is larger the larger are the fractions of contractible activities in South and the smaller are the fractions of contractible activities in North.

A key implication of this proposition is that lower contractual frictions in South encourage offshoring, while lower contractual frictions in North discourage offshoring.

Although our emphasis in this paper is on the roles played by contractual frictions, it is useful to note that two additional sectoral characteristics also affect the extent of foreign sourcing: productivity dispersion and headquarter intensity. As to productivity dispersion, Helpman, Melitz and Yeaple (2004) show that it varies substantially across sectors. We have a natural measure of dispersion, embodied in the shape parameter κ of the Pareto distribution; productivity dispersion is larger the smaller this parameter is. It is evident from the formula for the share of offshoring firms, $\sigma_O^S = \left(\underline{\Theta}/\Theta_O^N\right)^{\kappa}$, that this share is declining in κ. Therefore, offshoring is more prevalent in sectors with more productivity dispersion.

Next consider the impact of η_h on the extent of foreign sourcing. In Antràs and Helpman (2004) we found that offshoring is less prevalent in sectors with higher headquarter intensity. This is easily generalized in the current model, in which contractual frictions vary across inputs and countries. Suppose that contractual frictions vary across inputs but not across countries, i.e., $\mu_m^N = \mu_m^S = \mu_m$

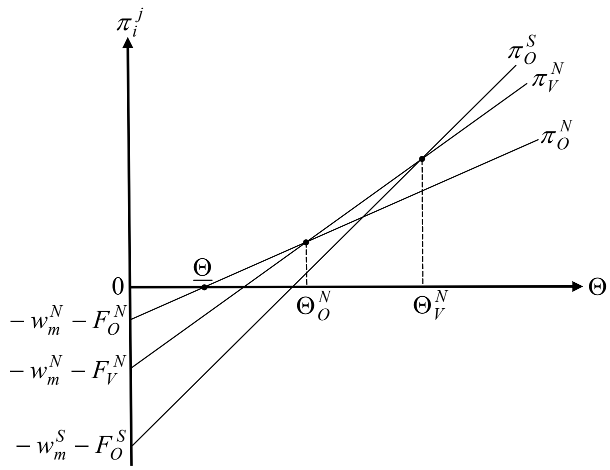

Figure 5: Profits from outsourcing and integration when $\eta_h \in (\eta_{hc}^N, \eta_{hc}^S)$ and $Z_O^S > Z_V^N$

and $\mu_h^N = \mu_h^S = \mu_h$, but μ_h differs from μ_m.[23] Then

$$\frac{Z_O^S}{Z_O^N} = \left(\frac{c_m^N}{c_m^S}\right)^{\alpha(1-\eta_h)}.$$

Since unit variable costs of components are lower in South, it follows that this ratio is lower in sectors that are more headquarter intensive. Moreover, since σ_O^S is increasing with the ratio Z_O^S/Z_O^N, we conclude that outsourcing in South is less prevalent in sectors that are more headquarter intensive.

A novel feature of the more complex model is that this result may not hold when contractual frictions vary across countries. To illustrate, consider the case in which $\mu_m^N = \mu_h^N = \mu_h^S = 1$ and $\mu_m^S = 0$. That is, there are no contractual frictions in North, there are no contractual frictions in headquarter services in South, but no activities of components are contractible in South. Moreover, let $\alpha = 0.4$, $\beta_{hO} = 0.2$, and $c_m^N/c_m^S = 1.1$. Then the ratio Z_O^S/Z_O^N is declining in η_h. However, replacing the revenue share $\beta_{hO} = 0.2$ with $\beta_{hO} = 0.5$ implies that the ratio Z_O^S/Z_O^N is increasing in η_h.

4.2 Medium Headquarter Intensity Sector

We next consider an industry with $\eta_h \in (\eta_{hc}^N, \eta_{hc}^S)$. Corollary 1 implies that in such an industry outsourcing dominates integration in South, because $Z_O^S > Z_V^S$, but in North integration dominates outsourcing for high-productivity firms, because $Z_V^N > Z_O^N$. Figure 5 presents the profit functions π_O^N and π_V^N, which describe the tradeoffs in the make-or-buy decision of firms who choose to manufacture components in North; the fixed costs are higher for integration while the marginal profits from higher productivity are also higher from integration. The figure shows a cutoff $\underline{\Theta}$ below which firms lose money either way, and therefore they do not produce. Firms with productivity

[23] Recall that in Antràs and Helpman (2004) $\mu_m^\ell = \mu_h^\ell = 0$ for $\ell = N, S$.

$\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ make higher profits from outsourcing, and firms with productivity above Θ_O^N make higher profits from integration. Naturally, if the fixed costs of integration are very low, such that $\Theta_O^N < \underline{\Theta}$, then some low-productivity firms choose not to produce and all those who produce integrate.

Figure 5 also shows a profit function from outsourcing in South, π_O^S.[24] The fixed costs of outsourcing in South are higher than the fixed costs of integration in North. As a result, integration in North dominates outsourcing in South for all Θ whenever $Z_O^S \leq Z_V^N$. In other words, if the impact on profitability of the less favorable contractual environment in South is large enough, relative to the South's unit cost advantage c_m^S/c_m^N, so as to yield $Z_O^S \leq Z_V^N$, then no firm in this industry offshores. Instead the figure shows profits from outsourcing in South for $Z_O^S > Z_V^N$, which means that the South's unit cost advantage is large relative to its disadvantage in the contractual environment. But the figure exhibits a case in which the ratio $Z_O^S/Z_V^N > 1$ is not too large, so that the cutoff, Θ_V^N, at which profits from integration in North just equal profits from outsourcing in South, is larger than Θ_O^N. In this event firms with productivity below $\underline{\Theta}$ do not produce, those with $\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ outsource in North, those with productivity $\Theta \in \left(\Theta_O^N, \Theta_V^N\right)$ integrate in North, and those with higher productivity levels outsource in South. Also note that if Z_O^S/Z_V^N is higher, so as to imply $\Theta_V^N \in \left(\underline{\Theta}, \Theta_O^N\right)$, then no firm integrates in North. In this case a cutoff exists between $\underline{\Theta}$ and Θ_O^N such that firms between $\underline{\Theta}$ and this cutoff outsource in North and firms above this cutoff outsource in South.

We now focus on the case depicted in Figure 5, in which all three organizational forms that are feasible for a headquarter intensity level of $\eta_h \in \left(\eta_{hc}^N, \eta_{hc}^S\right)$ coexist, i.e., outsourcing in North, integration in North, and outsourcing in South. We wish to study the prevalence of these organizational forms. The share of firms that outsource in South is

$$\sigma_O^S = \left(\frac{\underline{\Theta}}{\Theta_V^N}\right)^\kappa, \tag{16}$$

where $\underline{\Theta}$ is again given by (14) and

$$\Theta_V^N = \frac{\left(F_O^S + w_m^S\right) - \left(F_V^N + w_m^N\right)}{Z_O^S - Z_V^N}. \tag{17}$$

It follows that this share is larger the larger the ratio $\left(Z_O^S - Z_V^N\right)/Z_O^N$ is. From Proposition 1 we have that Z_O^S is increasing in μ_j^S for $j = h, m$, while Z_i^N is increasing in μ_j^N for $j = h, m$, $i = O, V$. In addition, the Zs corresponding to a particular sourcing location are not a function of the degree of contractibility in the other country. We can thus conclude that offshoring, which takes the form of outsourcing in our middle headquarter intensity sector, is more prevalent the better the contractual environment in South is and the worse the contractual environment in North is.

[24]Since outsourcing in South dominates integration in South, we do not show the profit function from integration in South.

The share of firms that outsource in North is

$$\sigma_O^N = 1 - \left(\frac{\underline{\Theta}}{\Theta_O^N}\right)^{\kappa}, \tag{18}$$

where

$$\Theta_O^N = \frac{F_V^N - F_O^N}{Z_V^N - Z_O^N}. \tag{19}$$

Evidently, this share does not depend on contractual frictions in South (μ_m^S or μ_h^S). As a result, the share of firms that integrate in North, given by

$$\sigma_V^N = \left(\frac{\underline{\Theta}}{\Theta_O^N}\right)^{\kappa} - \left(\frac{\underline{\Theta}}{\Theta_V^N}\right)^{\kappa}, \tag{20}$$

or simply $\sigma_V^N = 1 - \sigma_O^N - \sigma_O^S$, varies inversely with the share of firms that outsource in South and is thus decreasing in μ_m^S and μ_h^S.

The effect of Northern contracting institutions on the shares σ_O^N and σ_V^N is more complicated. First note that σ_O^N is decreasing in the ratio Z_V^N/Z_O^N, which in turn is increasing in μ_m^N and decreasing in μ_h^N. Hence, unlike in our low headquarter intensity sector, an improvement in contracting institutions in North does not always lead to more firms outsourcing in North. The nature of this contracting improvement in North is important for the direction of the effect: better contractibility in headquarter services leads to relatively more outsourcing in North, but better contractibility in components leads to relatively less outsourcing. Finally, because both σ_O^N and σ_O^S are decreasing in μ_m^N, we conclude that the share of firms integrating in North is higher the higher the contractibility of components is in North. Interestingly, however, an improvement in the contractibility of headquarter services in North, which reduces foreign offshoring, does not always lead to an increase in the share of firms integrating in North.[25]

We can summarize our results for the intermediate-headquarter-intensity sector as follows:

Proposition 8 *Consider an industry with $\eta_h \in \left(\eta_{hc}^N, \eta_{hc}^S\right)$. Then no firm integrates in South and there exists a cutoff $\underline{\Theta}$ given by (14) such that firms with productivity $\Theta < \underline{\Theta}$ do not produce. In addition, there exist two thresholds Θ_V^N and Θ_O^N, defined by (17) and (19), such that if $\Theta_V^N > \Theta_O^N > \underline{\Theta}$ then:*
(i) Firms with productivity $\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ outsource in North, those with productivity $\Theta \in \left(\Theta_O^N, \Theta_V^N\right)$ integrate in North, and firms with higher productivity outsource in South.
(ii) The fraction of offshoring firms is larger (where offshoring takes the form of outsourcing) and the fraction of firms that integrate in North is smaller the larger the fractions of contractible activities are in South. The fraction of firms that outsource in North is not affected by contractual frictions in South.
(iii) The fraction of offshoring firms is smaller and the fraction of firms that source in North is larger

[25] The reason is that this type of contracting improvement improves the profitability of integration in North relative to offshoring, but it reduces its profitability relative to outsourcing in North.

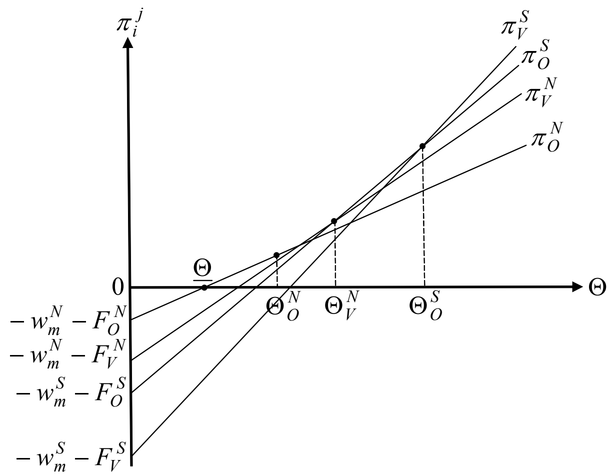

Figure 6: Profits from outsourcing and integration when $\eta_h > \eta_{hc}^S$: four organizational forms

the larger the fractions of contractible activities are in North. A disproportionate improvement in the contractibility of components in North may reduce however the share of firms that outsource in North, while a disproportionate improvement in the contractibility of headquarter services in North may reduce the share of firms that integrate in North.

As in the low headquarter intensity sector, we find in this case too that offshoring declines with contractual frictions in South and rises with contractual frictions in North. The main difference with the previous case is that the share of firms that outsource in North is now independent of contractibility in South and it no longer unambiguously increases when contacting institutions improve in North. Evidently, these differences stem from the fact that under the conditions of this proposition, offshoring competes with integration in North rather than with outsourcing in North.[26]

4.3 High Headquarter Intensity Sector

Propositions 7 and 8 imply that in sectors with headquarter intensity $\eta_h < \eta_{hc}^S$, no foreign direct investment takes place; there can be integration in North but not in South. It follows that offshoring via integration can emerge only in sectors with relatively high headquarter intensity. So consider a sector with $\eta_h > \eta_{hc}^S$. Corollary 1 implies that in such a sector the marginal profitability of integration is higher than the marginal profitability of outsourcing in each one of the countries, i.e., $Z_V^\ell > Z_O^\ell$ for $\ell = N, S$. In this case all four organizational forms may coexist in equilibrium: outsourcing in North, integration in North, outsourcing in South, and integration in South (FDI). This is illustrated in Figure 6. Firms with Θ below $\underline{\Theta}$ do not produce, those with $\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ outsource in North, those with $\Theta \in \left(\Theta_O^N, \Theta_V^N\right)$ integrate in North, those with $\Theta \in \left(\Theta_V^N, \Theta_O^S\right)$

[26] Note that as long as $Z_O^S > Z_V^N$ low enough fixed costs of outsourcing in South lead to an equilibrium in which low-productivity firms outsource in South and high-productivity firms outsource in North, with no firm integrating. In this type of equilibrium offshoring competes with outsourcing in North, just as it does in sectors with $\eta_h < \eta_{hc}^N$.

outsource in South, and those with $\Theta > \Theta_O^S$ integrate in South, i.e., they engage in foreign direct investment. Naturally, we can change the assumptions on fixed costs or the ranking of the marginal profits Z_i^j to eliminate one or more of the regimes in this case too. But their ranking by productivity will not be affected.

We next study the determinants of the relative prevalence of different organizational forms in an equilibrium in which all four forms coexist. Our first observation is that the shares of firms that outsource in North or integrate in North are given as before by (18) and (20), respectively. (Recall that these expressions were derived for intermediate headquarter intensity sectors.) Because the thresholds Θ_V^N and Θ_O^N are also defined as before, by (17) and (19), respectively, we conclude that the effects of changes in contractibility on the shares σ_O^N and σ_V^N are identical to those in sectors with $\eta_h \in \left(\eta_{hc}^N, \eta_{hc}^S\right)$, as summarized in Proposition 8.

It remains to discuss how the degree of contractibility of different inputs in different countries affects the relative prevalence of firms that outsource in South or engage in FDI there. A direct corollary of Proposition 8 is that the overall share of firms that offshore, given by

$$\sigma_O^S + \sigma_V^S = \left(\frac{\underline{\Theta}}{\Theta_V^N}\right)^\kappa, \tag{21}$$

is larger the larger contractibility is in South, and is lower the larger contractibility is in North.

How is the decrease in the share of firms that offshore distributed between firms that outsource and firms that engage in FDI when contractibility improves in North? To answer this question, note that

$$\sigma_V^S = \left(\frac{\underline{\Theta}}{\Theta_O^S}\right)^\kappa, \tag{22}$$

where $\underline{\Theta}$ is given by (14) and

$$\Theta_O^S = \frac{F_V^S - F_O^S}{Z_V^S - Z_O^S}. \tag{23}$$

It thus follows that the share σ_V^S is larger the larger the ratio $\left(Z_V^S - Z_O^S\right)/Z_O^N$ is. Evidently, the share of firms that do FDI falls as a result of increases in μ_j^N for $j = N, S$. Moreover, the fraction of offshoring firms that do FDI is given by

$$\frac{\sigma_V^S}{\sigma_O^S + \sigma_V^S} = \left(\frac{\Theta_V^N}{\Theta_O^S}\right)^\kappa, \tag{24}$$

which from (17) and (23) is an increasing function of $\left(Z_V^S - Z_O^S\right)/\left(Z_O^S - Z_V^N\right)$. We can thus conclude that an improvement of contractibility in North increases the prevalence of FDI relative to foreign outsourcing. Given that the share of firms engaged in FDI is negatively affected by such an improvement in contractibility in North, we also conclude that the share of firms that outsource falls. In sum, larger contractibility in North is associated with lower shares of both types of offshoring firms, with the decrease falling disproportionately on firms that outsource.

We noted above that an improvement in contracting institutions in South increases the share of firms that offshore. We next want to study the effects of this change on the relative prevalence of the two distinct types of offshoring: outsourcing and FDI. In doing so, it is important to distinguish between improvements in the contractibility of components and improvements in the contractibility of headquarter services.

Consider the former first. Remember that the share of firms that do FDI, σ_V^S, is increasing in the ratio $\left(Z_V^S - Z_O^S\right)/Z_O^N$, which we can write as $\left(Z_V^S/Z_O^S - 1\right)/\left(Z_O^N/Z_O^S\right)$. We have established above that both Z_O^S and the ratio Z_V^S/Z_O^S are increasing in μ_m^S. As a result, the share of firms that do FDI is increasing in the contractibility of components in South.

The effect of μ_m^S on σ_O^S is more complicated. From (21) and (22) we obtain

$$\sigma_O^S = \left(\frac{\underline{\Theta}}{\Theta_V^N}\right)^\kappa - \left(\frac{\underline{\Theta}}{\Theta_O^S}\right)^\kappa .$$

A larger contractibility of components in South makes foreign outsourcing relatively more profitable than integration in North ($\underline{\Theta}/\Theta_V^N$ falls), but it also decreases the profitability of outsourcing in South relative to FDI ($\underline{\Theta}/\Theta_O^S$ falls). The balance of these two effects is in general ambiguous, and we cannot rule out that increases in μ_m^S actually reduce the share of firms that outsource in South. Moreover, although the above discussion might have suggested that an improvement in the contractibility of components in South has a disproportionately large effect on FDI relative to outsourcing, it is possible to generate numerical examples in which the ratio σ_V^S/σ_O^S is actually decreasing in μ_m^S.[27]

We finally study the effects of an improvement in the contractibility of headquarter services in South on the share of firms that outsource in South and the share of firms that engage in FDI. As noted above, the fraction of offshoring firms that do FDI, $\sigma_V^S/\left(\sigma_V^S + \sigma_O^S\right)$, is an increasing function of $\left(Z_V^S - Z_O^S\right)/\left(Z_O^S - Z_V^N\right)$. Rewriting this expression as $\left(Z_V^S/Z_O^S - 1\right)/\left(1 - Z_V^N/Z_O^S\right)$, shows that it is decreasing in μ_h^S. As a result, we conclude that an improvement in the contractibility of headquarter services in South increases the share of firms offshoring there, with a disproportionately positive effect on the share of firms that outsource in South. As a matter of fact, the disproportionate effect may be large enough to generate a negative relationship between μ_h^S and the share of firms that engage in FDI.

The results we obtained for the high-headquarter-intensity sector can be summarized as follows

Proposition 9 *Consider an industry with $\eta_h > \eta_{hc}^S$. Then there exists a cutoff $\underline{\Theta}$ given by (14) such that firms with productivity $\Theta < \underline{\Theta}$ do not produce. In addition, there exist three thresholds Θ_V^N, Θ_O^N and Θ_O^S, defined by (17), (19) and (23), such that if $\Theta_V^N > \Theta_O^N > \underline{\Theta}$ then:*
(i) Firms with productivity $\Theta \in \left(\underline{\Theta}, \Theta_O^N\right)$ outsource in North, those with productivity $\Theta \in \left(\Theta_O^N, \Theta_V^N\right)$ integrate in North, those with productivity $\Theta \in \left(\Theta_V^N, \Theta_O^S\right)$ outsource in South, and firms with higher

[27] For example, assume that $\alpha = 0.5$, $c_h^N = c_m^N = 1$, $c_h^S = c_m^S = 0.7$, $\eta = 0.5$, $\beta_O = 0.3$, $\beta_V = 0.5$, $\mu_h^N = \mu_h^S = 0.3$, and $\mu_m^N = 1$. In this case $Z_V^S > Z_O^S > Z_V^N > Z_O^N$ for $\mu_m^S = 0.5$ and $\mu_m^S = 0.7$, and the ratio $\left(Z_V^S - Z_O^S\right)/\left(Z_O^S - Z_V^N\right)$ is lower when $\mu_m^S = 0.7$ than when $\mu_m^S = 0.5$. Hence, an increase in μ_m^S can reduce the fraction of offshoring firms that engage in FDI.

productivity integrate in South.
(ii) The fraction of offshoring firms is larger and the fraction of firms that integrate in North is smaller the larger are the fractions of contractible activities in South. The fraction of firms that outsource in North is not affected by contractual frictions in South. A disproportionate improvement in the contractibility of components in South may reduce the share of firms that outsource in South, while a disproportionate improvement in the contractibility of headquarter services in South may reduce the share of firms that integrate in South.
(iii) The fractions of offshoring firms, both offshore outsourcing firms and firms engaged in FDI, are smaller and the share of firms that source in North is larger the larger are the fractions of contractible activities in North. A disproportionate improvement in the contractibility of components in North may, however, reduce the share of firms that outsource in North, while a disproportionate improvement in the contractibility of headquarter services in North may reduce the share of firms that integrate in North. Moreover, the fraction of outsourcers among the set of offshoring firms is larger the smaller the fractions of contractible activities μ_h^N and μ_m^N in North are.

An important implication of this proposition is that a better contractual environment in South or a worse contractual environment in North do not equally encourage offshore outsourcing and FDI; they tend to encourage offshore outsourcing relatively more, except in cases in which the contractual improvement in South affects disproportionately the production of components.

5 Concluding Comments

In this paper we have generalized the global sourcing model of Antràs and Helpman (2004) to accommodate varying degrees of contractual frictions. In the model, a continuum of firms with heterogenous productivities decide whether to integrate or outsource intermediate inputs and in which countries to source the inputs. Final-good producers and their suppliers make relationship-specific investments which are only partially contractible, both in an integrated firm and in an arm's-length relationship. The degree of contractibility can vary across countries and inputs.

Our model generates equilibria in which firms with different productivity levels chose different ownership structures and supplier locations. Assuming a Pareto distribution of productivity, we studied the effects of changes in the quality of contractual institutions on the relative prevalence of these organizational forms. We have shown that an improvement in contractual institutions in South raises the prevalence of offshoring, but it can reduce the relative prevalence of either FDI or offshore outsourcing if it affects disproportionately the contractibility of headquarter services or components, respectively. This result embodies one of the major messages of the paper: the relative prevalence of alternative organizational forms depends not only on cross-country differences in contractibility, but also on the degree to which contractual institutions are biased toward inputs controlled by the final-good producer or other suppliers.

Although our model is partial equilibrium in scope, it can be embodied in a general equilibrium framework. Such an analysis might shed light on the sources of international income differences and their relationship to the structure of contractual frictions and the resulting trade and investment. Acemoglu, Antràs and Helpman (2006) provide a first step in this direction by analyzing the impact of contractual frictions on technology choice and the resulting productivity levels, but their model does not feature trade in intermediate inputs nor foreign direct investment. For this reason it cannot address the issues discussed in this paper. It is necessary to integrate the choice of technology with the choice of organizational form in order to obtain a unified theory which is suitable for the study of links between the quality of contractual institutions, productivity, and trade and investment.

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Appendix

A.1 Derivation of the Profit Function (8) and Proof of Lemma 1

We discuss in this appendix properties of the solution to the final-good producer's optimization problem

$$\max_{\{x_h(i)\}_{i=0}^1, \{x_m(i)\}_{i=0}^1} \pi \equiv R - c_h \int_0^1 x_h(i)\, di - c_m \int_0^1 x_m(i)\, di - w_m,$$

subject to the incentive compatibility constraints (4) and the revenue and output equations (1) and (2). First note from (3) that

$$R - \sum_{\ell=h,m} c_\ell (1-\mu_\ell) x_{\ell n} = \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right) R.$$

Therefore the maximization problem can be expressed as

$$\max_{\{x_h(i)\}_{i=0}^{\mu_h}, \{x_m(i)\}_{i=0}^{\mu_m}} \pi \equiv \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right) R - \sum_{\ell=h,m} c_\ell \int_0^{\mu_\ell} x_\ell(i)\, di - w_m, \tag{A1}$$

subject to

$$R = \left(K_1 \left[\exp \sum_{\ell=h,m} \alpha\eta_\ell \int_0^{\mu_\ell} \log x_\ell(i)\, di\right]\right)^{\frac{1}{1-\alpha\omega}}, \tag{A2}$$

where

$$K_1 = \alpha^{\alpha\omega} \theta^\alpha A^{1-\alpha} \eta_h^{-\alpha\eta_h} \eta_m^{-\alpha\eta_m} \left(\frac{\beta_m \eta_m}{c_m}\right)^{\alpha\omega_m} \left(\frac{\beta_h \eta_h}{c_h}\right)^{\alpha\omega_h}.$$

This representation of the revenue is obtained by substituting (4) into (2) and the result into (1). The first-order conditions (6) follow directly from this problem.

To prove Lemma 1, note from (3) and (6) that $x_{jc} > x_{jn}$ if and only if

$$\frac{1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell}{1 - \alpha\omega} > \beta_j.$$

But, since $\beta_\ell \in (0,1)$ for $\ell = h, m$, the left-hand side is larger than 1 while the right-hand side is smaller than 1, implying $x_{jc} > x_{jn}$.

Using the expression for R from (A2), the first-order conditions (6) can be expressed as

$$x_{jc} = \frac{1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell}{1 - \alpha\omega} \left(\frac{\eta_j}{c_j}\right) \alpha \left(K_1 x_{hc}^{\alpha\mu_h\eta_h} x_{mc}^{\alpha\mu_m\eta_m}\right)^{\frac{1}{1-\alpha\omega}}, \quad \text{for } j = h, m.$$

The solution to this system of equations yields (7).

Next, from (A1) we have

$$\pi = \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right) R - \sum_{\ell=h,m} c_\ell \int_0^{\mu_\ell} x_\ell (i)\, di - w_m.$$

Substituting (6) into this expression yields

$$\pi = (1-\alpha)\left(\frac{1-\alpha\sum_{\ell=h,m}\beta_\ell\omega_\ell}{1-\alpha\omega}\right) R - w_m.$$

Using (A2) together with (7) yields (8).

A.2 Proof of Proposition 1

To prove Proposition 1, first note from (8) that profits are trivially a decreasing function of input costs and the outside option of suppliers. To show that profits are decreasing in ω_j, $j = h, m$, requires more involved arguments. For this purpose first note that π is decreasing in ω_j if and only if

$$\pi_\omega \equiv \frac{\beta_m^{\alpha\omega_m}\beta_h^{\alpha\omega_h}\left(1-\alpha\sum_{\ell=h,m}\beta_\ell\omega_\ell\right)^{1-\alpha\omega}}{(1-\alpha\omega)^{1-\alpha\omega}}$$

is decreasing in ω_j. Taking logarithms of both sides and differentiating, we obtain

$$\frac{\partial \ln \pi_\omega}{\partial \omega_h} = \alpha \ln \beta_h - \alpha \ln\left(\frac{1-\alpha\sum_{\ell=h,m}\beta_\ell\omega_\ell}{1-\alpha\omega}\right) + \frac{\alpha\left[1-\alpha\beta_m\omega_m - \beta_h(1-\alpha\omega_m)\right]}{1-\alpha\sum_{\ell=h,m}\beta_\ell\omega_\ell}.$$

Moreover,

$$\frac{\partial^2 \ln \pi_\omega}{\partial \omega_h^2} = -\frac{\alpha^2\left[1-\alpha\beta_m\omega_m - \beta_h(1-\alpha\omega_m)\right]^2}{\left(1-\alpha\sum_{\ell=h,m}\beta_\ell\omega_\ell\right)^2(1-\alpha\omega)} < 0,$$

because

$$\begin{aligned} 1-\beta_m\alpha\omega_m - \beta_h(1-\alpha\omega_m) &\geq 1-\alpha\omega_m - \beta_h(1-\alpha\omega_m) \\ &= (1-\beta_m)(1-\alpha\omega_m) > 0. \end{aligned}$$

Therefore

$$\frac{\partial \ln \pi_\omega}{\partial \omega_h} < \left.\frac{\partial \ln \pi_\omega}{\partial \omega_h}\right|_{\omega_h=0} = g(\omega_m),$$

where

$$g(\omega_m) \equiv \alpha\ln\beta_h - \alpha\ln\left(\frac{1-\alpha\beta_m\omega_m}{1-\alpha\omega_m}\right) + \frac{\alpha\left[1-\alpha\beta_m\omega_m - \beta_h(1-\alpha\omega_m)\right]}{1-\alpha\beta_m\omega_m}.$$

Next note that

$$g'(\omega_m) = -\frac{\alpha^2(1-\beta_m)\left[1-\beta_m\alpha\omega_m - \beta_h(1-\alpha\omega_m)\right]}{(1-\alpha\beta_m\omega_m)^2(1-\alpha\omega_m)} < 0,$$

which implies

$$\frac{\partial \ln \pi_\omega}{\partial \omega_h} < g(0) = \alpha \ln \beta_h + \alpha (1 - \beta_h) < 0.$$

The last inequality results from the fact that $\ln \beta_h + 1 - \beta_h$ is maximized at $\beta_h = 1$, in which case $g(0)|_{\beta_h=1} = 0$. Yet $\beta_h \in (0,1)$, and therefore we have the inequality.

We have thus proved that profits are declining in ω_h, and therefore they are rising in μ_h (because $\omega_h = (1 - \mu_h)\eta_h$). Symmetric arguments show that profits are also declining in ω_m and therefore increasing in μ_m.

Note finally that the only channel through which the parameters ω_h and ω_m affect profits is through the function Z in (9). Hence, it is also the case that Z is increasing in μ_h and μ_m.

A.3 Characterization of β_j^* and Proof of Proposition 2

In order to characterize β_j^*, note that substituting $\beta_k = 1 - \beta_j$ into

$$\beta_m^{\alpha\omega_m} \beta_h^{\alpha\omega_h} \left(1 - \alpha \sum_{\ell=h,m} \beta_\ell \omega_\ell\right)^{1-\alpha\omega}$$

and computing the partial derivative with respect to β_j yields an expression which is proportional to the polynomial

$$(\omega_j - \omega_k)\beta_j^2 - 2\omega_j (1 - \alpha\omega_k)\beta_j + \omega_j (1 - \alpha\omega_k).$$

Equating this polynomial to 0 yields the solution (10). First note that this solution can be expressed as

$$\beta_j^* = \sqrt{\omega_j (1 - \alpha\omega_k)} \frac{\left[\sqrt{\omega_j (1 - \alpha\omega_k)} - \sqrt{\omega_k (1 - \alpha\omega_j)}\right]}{\omega_j - \omega_k} \quad \text{for } j,k = h,m, \quad k \neq j,$$

and that for $k \neq j$, $\sqrt{\omega_j (1 - \alpha\omega_k)} > \sqrt{\omega_k (1 - \alpha\omega_j)}$ if and only if $\omega_j > \omega_k$. Under the circumstances $\beta_j^* > 0$ for $j = h, m$, and since $\beta_h^* + \beta_m^* = 1$, it also implies $\beta_j^* < 1$. Second note that the second root of the quadratic equation from which β_j^* has been solved is

$$\beta_j^* = \frac{\omega_j (1 - \alpha\omega_k) + \sqrt{\omega_h \omega_m (1 - \alpha\omega_h)(1 - \alpha\omega_m)}}{\omega_j - \omega_k} \quad \text{for } j,k = h,m, \quad k \neq j,$$

and this root is larger than 1, because $\omega_j (1 - \alpha\omega_k) > (\omega_j - \omega_k)$. It follows that profits are rising with β_j for $0 < \beta_j < \beta_j^*$ and declining with β_j for $\beta_j^* < \beta_j < 1$.

To prove Proposition 2 first note that L'Hôpital's rule implies $\beta_j^* \to 1/2$ when $\omega_j \to \omega_k$ for $k \neq j$. Second,

$$\begin{aligned}
(\beta_h^* - \beta_m^*)(\omega_h - \omega_m) &= \omega_h (1 - \alpha\omega_m) + \omega_m (1 - \alpha\omega_h) - 2\sqrt{\omega_h \omega_m (1 - \alpha\omega_h)(1 - \alpha\omega_m)} \\
&= \left[\sqrt{\omega_h (1 - \alpha\omega_m)} - \sqrt{\omega_m (1 - \alpha\omega_h)}\right]^2 \geq 0, \qquad \text{(A3)}
\end{aligned}$$

with strict inequality holding when $\omega_h \neq \omega_m$, because in this case $\sqrt{\omega_h (1 - \alpha\omega_m)} \neq \sqrt{\omega_m (1 - \alpha\omega_h)}$.

Moreover,

$$\left[\sqrt{\omega_h\left(1-\alpha\omega_m\right)}-\sqrt{\omega_m\left(1-\alpha\omega_h\right)}\right]\left(\omega_h-\omega_m\right)>0 \quad \text{for} \;\; \omega_h\neq\omega_m.$$

Next differentiate (A3) to obtain

$$\frac{\partial\left(\beta_h^*-\beta_m^*\right)}{\partial\omega_h}=\left(\frac{\beta_h^*-\beta_m^*}{\omega_h-\omega_m}\right)\sqrt{\frac{\omega_m\left(1-\alpha\omega_m\right)}{\omega_h\left(1-\alpha\omega_h\right)}}.$$

The previous arguments then establish that the right-hand side is strictly positive. It follows that β_h^* is strictly increasing in ω_h. A symmetric argument implies that β_m^* is strictly increasing in ω_m. Therefore β_h^* is strictly declining in ω_m.

A.4 Determinants of the Make-or-Buy Decision and Proof of Proposition 3

To formally prove Proposition 3, we need to study the properties of the ratio Z_V/Z_O. From (9), we have

$$\frac{Z_V}{Z_O}=\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)^{\alpha\omega_m/(1-\alpha)}\left(\frac{\beta_{hV}}{\beta_{hO}}\right)^{\alpha\omega_h/(1-\alpha)}\left(\frac{1-\alpha\sum_{\ell=h,m}\beta_{\ell V}\omega_\ell}{1-\alpha\sum_{\ell=h,m}\beta_{\ell O}\omega_\ell}\right)^{(1-\alpha\omega)/(1-\alpha)}.$$

It is useful to work with the following monotonic transformation of Z_V/Z_O:

$$\begin{aligned}(1-\alpha)\ln\left(\frac{Z_V}{Z_O}\right) &= \alpha\omega_m\ln\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)+\alpha\omega_h\ln\left(\frac{\beta_{hV}}{\beta_{hO}}\right)\\ &\quad+\left(1-\alpha\left(\omega_h+\omega_m\right)\right)\ln\left(\frac{1-\alpha\left(\beta_{hV}\omega_h+\left(1-\beta_{hV}\right)\omega_m\right)}{1-\alpha\left(\beta_{hO}\omega_h+\left(1-\beta_{hO}\right)\omega_m\right)}\right).\end{aligned}\tag{25}$$

We will first prove that $\ln\left(Z_V/Z_O\right)$ is an increasing function of ω_h and a decreasing function of ω_m. This will immediately imply that Z_V/Z_O is increasing in η_h and μ_m and decreasing in η_m and μ_h.

Let us start with the effect of ω_m. Straightforward differentiation of (25) delivers

$$\begin{aligned}(1-\alpha)\frac{\partial\ln\left(Z_V/Z_O\right)}{\partial\omega_m} &= \alpha\ln\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)-\alpha\ln\left(\frac{1-\alpha\left(\beta_{hV}\omega_h+\left(1-\beta_{hV}\right)\omega_m\right)}{1-\alpha\left(\beta_{hO}\omega_h+\left(1-\beta_{hO}\right)\omega_m\right)}\right)\\ &\quad+\frac{\left(1-\alpha\omega_h\right)\alpha\left(1-\alpha\left(\omega_h+\omega_m\right)\right)\left(\beta_{hV}-\beta_{hO}\right)}{\left(1-\alpha\left(\beta_{hO}\omega_h+\left(1-\beta_{hO}\right)\omega_m\right)\right)\left(1-\alpha\left(\beta_{hV}\omega_h+\left(1-\beta_{hV}\right)\omega_m\right)\right)}.\end{aligned}$$

To show that $\partial\ln\left(Z_V/Z_O\right)/\partial\omega_m<0$ we will proceed in two steps. We will first show that $\partial^2\ln\left(Z_V/Z_O\right)/\partial\omega_m^2<0$ and then that, when evaluated at $\omega=0$, $\partial\ln\left(Z_V/Z_O\right)/\partial\omega_m$ is negative.

Step 1: Simple though cumbersome differentiation delivers

$$\begin{aligned}(1-\alpha)\frac{\partial^2 \ln(Z_V/Z_O)}{\partial\omega_m^2} &= -\frac{(1-\alpha\omega_h)(\beta_{hV}-\beta_{hO})\alpha^2}{(1-\alpha(\beta_{hO}\omega_h+(1-\beta_{hO})\omega_m))^2(1-\alpha(\beta_{hV}\omega_h+(1-\beta_{hV})\omega_m))^2}\\ &\quad\times g(\alpha,\beta_{hV},\beta_{hO},\omega_h,\omega_m),\end{aligned}$$

where

$$\begin{aligned}g(\alpha,\beta_{hV},\beta_{hO},\omega_h,\omega_m) &= \beta_{hV}+\beta_{hO}+2\alpha\omega_h-\beta_{hV}\alpha\omega_m-2\beta_{hV}\alpha\omega_h-\alpha\beta_{hO}\omega_m-2\alpha\beta_{hO}\omega_h\\ &\quad+2\beta_{hV}\alpha\beta_{hO}\omega_m-2\beta_{hV}\alpha\beta_{hO}\omega_h-2\alpha^2\omega_m\omega_h+3\beta_{hV}\alpha^2\omega_m\omega_h\\ &\quad+3\alpha^2\beta_{hO}\omega_m\omega_h-4\beta_{hV}\alpha^2\beta_{hO}\omega_m\omega_h-\beta_{hV}\alpha^2(\omega_h)^2-\alpha^2\beta_{hO}(\omega_h)^2\\ &\quad+4\beta_{hV}\alpha^2\beta_{hO}(\omega_h)^2.\end{aligned}$$

The function $g(\cdot)$ is somewhat complex, but we can show that it only takes positive values in the relevant domain. To see this note that

$$\frac{\partial g(\cdot)}{\partial\omega_m} = -\alpha(\beta_{hV}(1-\alpha\omega_h)(1-\beta_{hO})+\beta_{hO}(1-\beta_{hV})(1-\alpha\omega_h)+2\alpha\omega_h(1-\beta_{hO})(1-\beta_{hV}))<0,$$

and thus it suffices to check that $g(\alpha,\beta_{hV},\beta_{hO},\omega_h,1)$ is positive. But this follows from

$$\begin{aligned}g(\alpha,\beta_{hV},\beta_{hO},\omega_h,1) &= (1-\alpha)(\beta_{hV}-\beta_{hO}+2\beta_{hO}(1-\alpha\omega_h)+2\alpha\omega_h(1-\beta_{hV}))\\ &\quad+(2\beta_{hV}\beta_{hO}(1-\alpha\omega_h)+\beta_{hV}\alpha\omega_h(1-\beta_{hO})+\omega_h\beta_{hO}\alpha(1-\beta_{hV}))\alpha(1-\omega_h),\end{aligned}$$

which indeed is a sum of positive terms.

Step 2: Next we note that, when evaluated at $\omega_m=0$, we have that $\partial\ln(Z_V/Z_O)/\partial\omega_m<0$ if and only if

$$h(\beta_{hO})=\alpha\ln\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)-\alpha\ln\left(\frac{1-\alpha\beta_{hV}\omega_h}{1-\alpha\beta_{hO}\omega_h}\right)+\frac{\alpha(1-\alpha\omega_h)^2(\beta_{hV}-\beta_{hO})}{(1-\alpha\beta_{hO}\omega_h)(1-\alpha\beta_{hV}\omega_h)}<0.$$

But note that

$$h'(\beta_{hO})=\frac{\alpha(1-\alpha\omega_h)(\alpha\omega_h(1-\beta_{hO})+\beta_{hO}(1-\alpha\omega_h))}{(1-\beta_{hO})(1-\alpha\beta_{hO}\omega_h)^2}>0,$$

and thus $\beta_{hV}=\arg\sup h(\beta_{hO})$ (remember that $\beta_{hO}\geq\beta_{hV}$ is not possible). Finally, note that $h(\beta_{hV})=0$, and thus it follows that $h(\beta_{hO})<0$ and $\partial\ln(Z_V/Z_O)/\partial\omega_m<0$ for all $\omega_m\in(0,1)$. This completes the proof that Z_V/Z_O is an decreasing function of ω_m.

The proof of $\partial\ln(Z_V/Z_O)/\partial\omega_h>0$ is analogous, though we need not repeat all the steps. It suffices to note that letting $\hat{\beta}_{hV} = 1 - \beta_{hV}$ and $\hat{\beta}_{hO} = 1 - \beta_{hO}$, we can write (25) as

$$\begin{aligned}(1-\alpha)\ln\left(\frac{Z_V}{Z_O}\right) &= -\alpha\omega_m \ln\left(\frac{\hat{\beta}_{hO}}{\hat{\beta}_{hV}}\right) - \alpha\omega_h \ln\left(\frac{1-\hat{\beta}_{hO}}{1-\hat{\beta}_{hV}}\right) \\ &\quad - (1-\alpha(\omega_h+\omega_m))\ln\left(\frac{1-\alpha\left(\left(1-\hat{\beta}_{hO}\right)\omega_h + \hat{\beta}_{hO}\omega_m\right)}{1-\alpha\left(\left(1-\hat{\beta}_{hV}\right)\omega_h + \hat{\beta}_{hV}\omega_m\right)}\right),\end{aligned}$$

where $\hat{\beta}_{hO} > \hat{\beta}_{hV}$. Notice that this expression is analogous to the one we used to prove that $\partial \ln(Z_V/Z_O)/\partial\omega_m < 0$, but with negative signs throughout. We can thus conclude that Z_V/Z_O is increasing in ω_h.

Given these results we can conclude that Z_V/Z_O is increasing in η_h and μ_m and decreasing in η_m. Next, we want to show that outsourcing is preferred to integration for a low enough η_h, while the converse is true for a high enough η_h. This follows from noting that when $\eta_h \to 0$, then $\omega_h \to 0$ and

$$(1-\alpha)\ln\left(\frac{Z_V}{Z_O}\right) \to \alpha\omega_m \ln\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right) + (1-\alpha\omega_m)\ln\left(\frac{1-(1-\beta_{hV})\alpha\omega_m}{1-(1-\beta_{hO})\alpha\omega_m}\right).$$

Because $\beta_{hV} > \beta_{hO}$, using the fact that $(1-ax)x^{a/(1-a)}$ is an increasing function of x for $a \in (0,1)$ and $x \in (0,1)$, we can conclude that

$$\left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)^{\alpha\omega_m/(1-\alpha\omega_m)} \left(\frac{1-(1-\beta_{hV})\alpha\omega_m}{1-(1-\beta_{hO})\alpha\omega_m}\right) < 1.$$

This implies that $Z_V/Z_O < 1$ and thus profits are higher under outsourcing in such a case (remember that fixed and variable costs are here assumed identical under integration and outsourcing). Similarly, when $\eta_h \to 1$, then $\omega_m \to 0$ and

$$(1-\alpha)\ln\left(\frac{Z_V}{Z_O}\right) \to \alpha\omega_h \ln\left(\frac{\beta_{hV}}{\beta_{hO}}\right) + (1-\alpha\omega_h)\ln\left(\frac{1-\alpha\beta_{hV}\omega_h}{1-\alpha\beta_{hO}\omega_h}\right).$$

Again, using the fact that $(1-ax)x^{a/(1-a)}$ is an increasing function of x for $a \in (0,1)$ and $x \in (0,1)$, we can conclude that $\ln(Z_V/Z_O) > 0$ and profits are higher under integration in such a case.

Given the monotonicity of $\ln(Z_V/Z_O)$ and the two extreme cases $\eta_h \to 0$ and $\eta_h \to 1$, we can thus conclude that a unique headquarter- intensity cutoff $\eta_{hc} \in (0,1)$ exists, such that profits are higher under outsourcing for $\eta_h < \eta_{hc}$ and higher under integration for $\eta_h > \eta_{hc}$.

To prove part (ii) of Proposition 3 it suffices to use the implicit function theorem. The cutoff η_{hc} is implicitly defined by $Z_V/Z_O = 1$. Since Z_V/Z_O is increasing in η_h, decreasing in μ_h, and increasing in μ_m, we can conclude that the cutoff η_{hc} is higher the larger μ_h is and the smaller μ_m is.

A.5 Numerical Example in Footnote 19

Remember that the ratio Z_V/Z_O is given by

$$\frac{Z_V}{Z_O} = \left(\frac{1-\beta_{hV}}{1-\beta_{hO}}\right)^{\alpha\omega_m/(1-\alpha)} \left(\frac{\beta_{hV}}{\beta_{hO}}\right)^{\alpha\omega_h/(1-\alpha)} \left(\frac{1-\alpha\sum_{\ell=h,m}\beta_{\ell V}\omega_\ell}{1-\alpha\sum_{\ell=h,m}\beta_{\ell O}\omega_\ell}\right)^{(1-\alpha\omega)/(1-\alpha)}.$$

Let $\alpha = 4/5$, $\beta_{hV} = 1/2$, $\beta_{hO} = 1/3$, and $\mu_h = \mu_m = \lambda = 1/4$. Then we have that, when $\eta_h = 0.4$, we obtain $Z_V/Z_O = 1.027$, while when $\eta_h = 0.5$, we have $Z_V/Z_O = 1.193$. If we raise λ to $1/2$, we instead obtain $Z_V/Z_O = 1.03$ when $\eta_h = 0.4$, and $Z_V/Z_O = 1.125$ when $\eta_h = 0.5$. Hence, the effect of λ (and thus overall contractibility) on the ratio Z_V/Z_O is ambiguous.