PARADIGM UNCERTAINTY AND THE ROLE OF MONETARY DEVELOPMENTS IN MONETARY POLICY RULES

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Abstract *When taking monetary policy decisions, central banks face considerable uncertainty about the transmission mechanism of monetary policy to the price level. In particular, the role played by monetary developments in the transmission mechanism is not well understood. Two paradigms exist: one assigns monetary developments an entirely passive role; the other gives money an active role, beyond that of an indicator variable. Taking such uncertainty as a starting point for analysis, this paper evaluates a number of monetary policy rules for short-term interest rate decisions in the face of paradigm uncertainty. It describes what constitutes an efficient rule in this context and discusses procedures leading to the adoption of such rules.*

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1. Introduction[1]

Uncertainty exists regarding the correct specification of structural macroeconomic models. Policy makers should factor their limited knowledge of the economy into the process by which they take monetary policy decisions. Otherwise, a monetary policy rule which is optimal (or near optimal) within one specific, favoured model might produce undesirable results in practice if that model is a poor description of the way the economy actually behaves.

This paper analyses a situation where uncertainty exists about the role played by monetary developments in the transmission mechanism of monetary policy to the price level. Broadly speaking, two main paradigms regarding the determination of the price level and the channels of monetary transmission can be identified in the literature. Under one paradigm, price developments are viewed as being driven by the interaction of aggregate demand and supply and cost pressures. Monetary policy influences price developments through its impact on demand conditions. Under the other paradigm, price developments are seen as primarily a monetary phenomenon. In particular, monetary policy influences prices and inflation via monetary developments.

Because of uncertainty about which paradigm best reflects the true (but unknown) structure of the economy, formulating a unified reference model is difficult. This difficulty is exacerbated if the available empirical evidence supports both paradigms and does not allow one model to be discarded in favour of the other (i.e. the testable restrictions implied by the models are not nested and both sets of restrictions fail to be rejected).[2] Agreeing on a single reference model might be especially difficult in a collegial decision-making body. With these concerns in mind, we investigate a situation where policy-makers are assumed to entertain two different reference models as guides for monetary policy decisions. Under these circumstances, we analyse how policy makers might formulate a monetary policy rule that "performs well" in both paradigms.

Several contributions to the literature have addressed the design of policy rules that "perform well" across various models. One strand of research (e.g., McCallum, 1988; Taylor, 1998; Blinder, 1998) is based on simulating different policy rules in a variety of models in order to uncover the characteristics that a rule should display in order to guarantee a good performance (in terms of moments) across models. Another approach (e.g., Levin, et al., 1999, 2001) is based on minimising an average (quadratic) loss function across different models.[3] From a Bayesian perspective, this approach corresponds to the adoption of flat or uniform priors about which model describes the "true" structure of the economy. In addition, when uncertainty is modeled within a unified reference model rather than across models, several papers (e.g., Sack, 1998; Martin and Salmon, 1999) have analysed the effects of Brainard (1967) uncertainty on the optimal monetary policy rule. More recently, the engineer literature on robust control has been applied to the implementation of monetary policy under uncertainty (e.g., Stock and Onatski, 1998; Tetlow and von zur Muehlen, 2000). Such analysis has shown how to incorporate structured and unstructured uncertainty into a unified reference model of the economy and illustrated how a solution of the resulting problem can be characterized in terms of finding a rule that performs well even in the worse case scenario.

This paper follows the spirit of Levin, et al. (1999, 2001). However, our approach differs in two respects. First, we focus on models that postulate completely different transmission mechanisms (i.e., models that represent different paradigms). Second, we also analyse rules for monetary policy that protect against the worse case scenario.

The paper is organised as follows. In Section 2, we briefly outline the two simple macroeconomic models used to define the essence of the two paradigms of monetary transmission. Section 3 motivates the approaches adopted in the paper, while Section 4 presents results for a variety of different policy rules. Section 5 briefly concludes.

2. Two models of the inflation process

This section describes two familiar models of the inflation process. Each model captures the essence of one of the two main paradigms of the monetary policy transmission mechanism, as discussed by Selody (2001).

2.1 The output gap model

The output gap model (henceforth OGM) relates inflation dynamics to deviations of output from its potential level. As the output gap widens, demand pressures increase. Demand conditions allow productive firms to widen their profit margins, thus creating inflationary pressure. This model has become the workhorse for much recent analysis of monetary policy. Its microeconomic foundations, relying on staggered price and wage adjustment (of the type suggested by Taylor (1980), Calvo (1983) and Rotemberg (1982)), have been developed in some detail.

Within the OGM, monetary policy (represented by a short-term nominal interest rate under the control of the central bank) influences price dynamics as follows. Since prices exhibit some stickiness, an increase in the nominal interest rate translates into higher real interest rates. Through intertemporal substitution, higher real interest rates constrain consumption and investment demand. By changing the short-term nominal interest rate, the central bank can influence demand conditions, the output gap and, via the Phillips curve, inflation.

In its simplest, backward-looking form, the model can thus be presented as:

$$y_t = l_{OGM}\, y_{t-1} - d_{OGM}\,(i_{t-1} - E_{t-1}\Delta p_t) + e_{s,t} \qquad (1)$$

$$\Delta p_t = \Delta p_{t-1} + b\,(y_{t-1} - y^*_{t-1}) + e_{s,t} \qquad (2)$$

$$y^*_t = 0 \qquad (3)$$

where y is output; y^* is potential output; i is the short-term nominal interest rate under the control of the central bank; p is the price level and e_d and e_s are demand and supply shocks respectively. $E_{t-1}x_t$ represents the expectation of x_t at time t-1, where t is (discrete) time. For notational simplicity, the variables are demeaned and detrended, such that potential output is zero (as reflected in equation (3)).

In this paper, we use a backward-looking version of the Phillips curve, rather than the forward-looking version which has found favour in the New Keynesian literature.[4] Like Rudebusch and Svensson (1998), we adopt this approach to facilitate the presentation of simple and transparent solution techniques to the model. The results should therefore be regarded as illustrative rather than empirically motivated.

We append a money demand equation to the basic OGM. As price and output dynamics are fully determined by equations (1) through (3), money does not play any active role in the transmission mechanism of monetary policy within the OGM. This money demand equation has a standard error correction specification, as shown below.

$$\Delta(m-p)_t \;=\; f\,\Delta(m-p)_{t-1} \;-\; J\,((m-p)_{t-1} \;-\; y_{t-1} \;+\; g\; i_{t-1}) \;+\; e_{m,t} \qquad (4)$$

where m is the money stock and e_m is a monetary shock.

Some simple algebra allows the OGM to be written in a state space form.[5]

$$\begin{bmatrix} y_{t+1} \\ \Delta p_{t+1} \\ (m-p)_{t+1} \\ (m-p)_t \end{bmatrix} = \begin{bmatrix} (l_{OGM}+d_{OGM}b) & d & 0 & 0 \\ b & 1 & 0 & 0 \\ J & 0 & (1+f-J) & -f \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ \Delta p_t \\ (m-p)_t \\ (m-p)_{t-1} \end{bmatrix} + \begin{bmatrix} -d \\ 0 \\ -Jg \\ 0 \end{bmatrix} i_t + \begin{bmatrix} e_{d,t+1} \\ e_{s,t+1} \\ e_{m,t+1} \\ 0 \end{bmatrix}$$

which for simplicity we can denote:

$$\mathbf{x}_{t+1} \;=\; \mathbf{A}_{OG}\,\mathbf{x}_t \;+\; \mathbf{B}_{OG}\,\mathbf{i}_t \;+\; e_{OG,t+1} \;; \qquad E\, e_{OG}' e_{OG} \;=\; S_{OG} \qquad (5)$$

2.2 *The P-star model*

The P-star model (henceforth P*) – originally proposed by Hallman, et al. (1991) and recently applied to the euro area by Gerlach and Svensson (2000) – gives an active role to monetary developments in inflation dynamics. Specifically, inflationary pressure is related to the real money gap, which measures the excess of real money over that consistent with monetary equilibrium. Prices revert to the level implied by monetary equilibrium through an error correction process.

The microeconomic foundations of the P* model are not well developed. Implicitly, the model can be related to the view that monetary disequilibria result in greater spending and thus more demand pressures (e.g. Laidler, 1998). More generally, the P* model can be seen as a reduced form representing a narrative view of monetary transmission with a long pedigree, namely one that places imperfections in the financial system, monetary expansions and the resulting credit booms and busts at the heart of explanations of macroeconomic developments (e.g. Friedman and Schwartz, 1962; Kindleberger, 1987; Minksy, 1982).

In the P* model, we continue to use the short-term nominal interest rate as the instrument of monetary policy. Monetary policy therefore affects inflation dynamics through influencing monetary developments. Continuing to characterize monetary policy using short-term interest rates despite the inherently monetary nature of the P* model permits comparisons with alternative models to be made.

We characterise the P* model following Gerlach and Svensson (2000).

$$y_t = l_{P*}\, y_{t-1} - d_{P*}\,(i_{t-1} - E_{t-1}\Delta p_t) + e_{s,t} \quad (6)$$

$$\Delta(m-p)_t = f\,\Delta(m-p)_{t-1} - J\,((m-p)_{t-1} - y_{t-1} + g\, i_{t-1}) + e_{m,t} \quad (7)$$

$$\Delta p_t = (1-w)\,\Delta p_{t-1} + w\,\Delta p^*_{t-1} - m\,(p_{t-1} - p^*_{t-1}) + e_{s,t} \quad (8)$$

$$p^*_t = m_t - y^*_t - m\, i^* = m_t \quad (9)$$

where the notation is the same as above, with i* the nominal short-term interest rate holding in steady state equilibrium with price stability, normalized to zero.

Again some simple algebra allows this system to be written in state space form as:

$$\begin{bmatrix} y_{t+1} \\ \Delta p_{t+1} \\ (m-p)_{t+1} \\ (m-p)_t \end{bmatrix} = \begin{bmatrix} l_{P*} & d_{P*} & d_{P*}(m+w) & -d_{P*}w \\ 0 & 1 & (m+w) & w \\ J & 0 & (1+f-J) & -f \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_t \\ \Delta p_t \\ (m-p)_t \\ (m-p)_{t-1} \end{bmatrix} + \begin{bmatrix} -d \\ 0 \\ -Jg \\ 0 \end{bmatrix} i_t + \begin{bmatrix} e_{d,t+1} \\ e_{s,t+1} \\ e_{md,t+1} \\ 0 \end{bmatrix}$$

or, in simplified notation, as:

$$\mathbf{x}_{t+1} = \mathbf{A}_{P*}\,\mathbf{x}_t + \mathbf{B}_{P*}\, i_t + e_{P*,t+1} \qquad E\, e_{P*}' e_{P*} = S_{P*} \quad (10)$$

Note that expression (10) does not have a block recursive structure (i.e. the upper right elements of the matrix $\mathbf{A}_{P*}$ are non-zero). This reflects the active role of money in the transmission of monetary policy actions to the price level.

2.3 *Some observations on the structure of the two models*

On the basis of the preceding discussion, two observations regarding the structure of the two models should be noted, since they have important implications for the empirical implementation of the approach suggested in this paper.

First, because of the inflation expectations term in the real interest rate, a number of cross equation restrictions are imposed by the two models. These restrictions imply that the aggregate demand and supply equation in the OGM - and all the three equations in the P* - should be estimated as a system rather than equation-by-equation. As a result, the parameter estimates for the P* model may differ from those estimated for the OGM, even if the structure of the equations is the same (as is the case for the aggregate demand equation (1) and (6)).

Second, the restrictions imposed by the two structural models on the unconstrained system:

$$\mathbf{x}_{t+1} \quad = \quad \mathbf{A}\,\mathbf{x}_t \quad + \quad \mathbf{B}\ \mathbf{i}_t \quad + \quad \mathrm{e}_{t+1} \qquad (11)$$

are not nested. In other words, using specification (11) as an encompassing model, the data may fail to reject *both* the OGM and P* models. In particular, excluding the money gap from the inflation equation (i.e. imposing $a_{23} = a_{24} = 0$ in (11), where a_{ij} is the element in the i th row and j th column of matrix A) is not sufficient to reject the P* model in favour of the OGM.

This second observation is crucial. It provides a basis for taking the view that making a choice between the two competing models of the inflation process may not be possible using the available data. The empirical plausibility of both models is central to the multi-model approach underlying this paper. If one of the models were to be rejected by the data while the other was not, the empirical basis for such a multi-model approach would not exist.

2.4 *Deriving optimal monetary policy rules for the two models*

To provide a metric for the comparison of competing monetary policy rules and to characterize optimal monetary policy in the context of the two models considered here, the central bank's objective function needs to be specified. Following much of the existing academic literature, we use a quadratic loss function which is a weighted average of the variance of the output gap and the variance of inflation around its target level consistent with price stability. The central bank's loss is then given by:

$$L \quad = \quad E_0 \left\{ y \sum_{i=0}^{\infty} (\Delta p_i)^2 \quad + \quad (1-y) \sum_{i=0}^{\infty} (y_i - y_i^*)^2 \right\} \qquad (12)$$

where the inflation objective has been normalized to zero. This expression can be written more concisely as:

$$L \quad = \quad E\,(\,\mathbf{x}'\,\mathbf{R}\,\mathbf{x}\,); \qquad \mathbf{R} \;=\; \begin{bmatrix} (1-y) & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Note that this objective function accords no weight to monetary dynamics or interest rate volatility. Developments in monetary growth or interest rates are ultimately only of concern to the central bank insofar as they affect developments in the goal variables, inflation and output. Central bank responses to current or lagged monetary developments therefore reflect the information such developments contain about future inflation and output gap developments. They do not imply an intrinsic concern for monetary dynamics.

The monetary policy problem facing the central bank in each of the two models can then be expressed as follows:

$$\textit{minimize}_g\; L(\mathbf{x}) \textit{ subject to } \mathbf{x}_{t+1} \;=\; \mathbf{A}_i\,\mathbf{x}_t \;+\; \mathbf{B}_i\; i_t \;+\; \mathrm{e}_{i,t+1} \qquad i \;=\; \mathrm{OG, P^*} \tag{13}$$

where g is a monetary policy rule. Given the simple backward-looking nature of the two models being considered, standard optimal control techniques can be used to solve this problem for the optimal monetary policy rule. These rules express the short-term interest rate as a function of the state vector, $\mathbf{x}_t$. In this simple linear quadratic framework, the optimal rules are linear and are denoted $i_t = f_{\mathrm{OG}}\,\mathbf{x}_t$ and $i_t = f_{\mathrm{P^*}}\,\mathbf{x}_t$ for the OGM and P* model respectively.

In this paper, we use parameter estimates from the literature to calibrate the two structural models. The selected parameter values are reported in Table 1. (Note that, notwithstanding the comments made above, we assume the same parameter value for the two models when a common parameter appears.) Table 2 reports the optimal policy rules for each of the two models when the loss function parameter $y = 0.5$.

As expected, the passive money paradigm (represented by the OGM) implies that monetary policy should not respond to monetary developments, whereas the active money paradigm (here captured by the P* model) implies the opposite. However, it is difficult to interpret the magnitude of the coefficients in the optimal rules given that the models are very simple. To get an intuition of the characteristics of the two models and their associated optimal rules we derive impulse response functions and Taylor curves.

2.5 *Characteristics of the two models*

a) *Impulse responses*

Some features of the two models are revealed by deriving the impulse response functions, which trace how each variable in the system responds to structural shocks. These functions naturally depend on how monetary policy responds to the shocks. The impulse responses are shown in Figure 1, where monetary growth is derived from the path of the real money stock and inflation dynamics. In these charts, short-term interest rates are assumed to follow the path prescribed by the optimal monetary policy rule for the model under investigation, derived as described above.

A number of features of the impulse responses are noteworthy.

First, the dynamic response of each system is shock specific. A demand shock implies different inflation, output and monetary dynamics from a supply shock. As a result, the monetary policy response varies with the nature of the shock.

Second, the responses to structural shocks are model specific. A demand shock produces one set of inflation, output and monetary dynamics in the OGM and another in the P* model. Reflecting the different model specifications, the optimal monetary policy response to a demand shock therefore also differs across models.

Third (and a special case of the preceding observation), in the OGM money demand shocks constitute “pure noise” in the sense that they do not affect either of the goal variables, inflation and output. As reflected in the optimal policy rule for the OGM, a well-designed monetary policy would therefore not respond to monetary developments. In contrast, in the P* model monetary shocks affect inflation and output developments: the former directly (since money enters the inflation equation via the P* terms) and the latter indirectly (through the effect of money on inflation expectations and thus real interest rates). As a result, optimal monetary policy will respond to monetary dynamics (resulting in a third channel of transmission from money to goal variables through changes in the short-term nominal interest rate).

This third observation characterizes more precisely the distinction between the passive view of monetary developments in the inflation process represented by OGM and the active view inherent to the P* model.

b) *Taylor curves*

Other features of the two models are revealed by Taylor curves, which trace out the locus of optimal inflation volatility / output volatility combinations that are attainable within each model. Each point on the Taylor curve is associated with an optimal monetary policy rule derived from problem (13) as the weight on inflation volatility in the loss function (γ) varies between zero and one. The closer the Taylor curve lies to the origin, the greater the scope for policy makers to stabilize inflation and output. The greater the convexity of the Taylor curve, the less pronounced the trade off between inflation volatility and output volatility.

Figures 2a and 2b show the Taylor curves for the OGM and P* model respectively, together with the point on the curve corresponding to the optimal rules with $y = 0.5$ derived above.

As one would expect, the Taylor curves differ, reflecting the different underlying structure of the two models. In particular, as shown in Figure 2c, the P* Taylor curve lies outside the OGM Taylor curve. This reflects the greater difficulty in stabilizing inflation and output in the P* model arising from the fundamental structure of the model and the shocks to it. In subsequent sections, it will prove useful to recall that stabilization in the P* model poses a greater challenge for monetary policy.

Note that a change in central bank preferences (y) represents a move along the Taylor curve, whereas a change in model parameters ($\mathbf{A}_i$, $\mathbf{B}_i$, S_i; i = OG, P*) in general results in a shift of the Taylor curve or a change in its form. However, because of the passive role played by money in inflation and output dynamics within the OGM, changes to the parameters of the money demand equation (4) (f, u, g) or to the variance of monetary shocks ($E\, e_m^2$) do not shift the Taylor curve in the OGM. In contrast, such changes do shift the Taylor curve in the P* model, a reflection of the active role of money in that context.

3. Designing monetary policy rules under paradigm uncertainty

3.1 Motivation

Given that neither the OGM nor the P* can be dismissed ex ante on empirical grounds, monetary policy decisions should allow scope for analysis and insights from both paradigms to play a role. In other words, the central bank should adopt a monetary policy strategy that "performs well" in both models, rather than being performing optimally in one model or in the other. Note that the characterization of the monetary policy problem facing central banks in expression (13) does not resolve the issue of how to select such a rule. This characterization of policy makers' preferences assumes that policy makers will minimize the loss function within a single structural model. It is silent as to how the implications of having two models, neither of which can be rejected on empirical grounds, should be addressed.

The problem of designing policy with rival models was analysed by Chow (1976) who suggested to construct a payoff matrix which shows the costs of basing the optimal policy on one of the models and assuming that the real world is represented by the rival model. The optimal policy under model uncertainty should thus be based on choosing the policy that causes least damage. The shortcoming of this approach is that the optimal policy under uncertainty is restricted to be either the optimal policy of one model or that of the other.

A simple example helps to illustrates this point. Consider a central bank that operated solely on the basis of the P* model. Given preferences y, this central bank would set short-term interest rates on the basis of the optimal rule for the P* model and thus achieve a point on the P* Taylor curve (as illustrated in Figure 2b). However, if economic outcomes were determined by the OGM, the adoption of this rule would be sub-optimal. In principle, such a rule may not stabilize the economy, resulting in the loss of price stability and explosive inflation. In less extreme cases (and in the examples shown here, as illustrated in Figure 3a), the rule would yield an inefficient inflation volatility / output volatility combination (i.e., one inside the OGM Taylor curve).

For example, for preferences y, any rule which yields an inflation volatility / output volatility combination in the area ABCD in Figure 3a would dominate the optimal P* rule, in the sense that it results in a lower loss according to the loss function (12). Indeed, any rule giving an inflation volatility / output volatility combination in the area AEF would strongly dominate the optimal P* rule, i.e. result in both lower inflation volatility and lower output volatility. Note the rules that are not fully optimal in the OGM (i.e., rules which do not lie on the OGM Taylor curve) can satisfy one or both of these criteria.

Of course, the mirror image of this example is also feasible. Consider a central bank that bases its monetary policy decisions on the OGM when economic outcomes are determined by the P* model. Again, sub-optimal policy will result. Figure 3b illustrates how any policy rule which yields an inflation volatility / output volatility combination in the area ABCD will dominate the optimal OGM rule in the P* model.

Therefore, monetary policy under paradigm uncertainty should be based on analysis using *both* models. A well-designed rule under paradigm uncertainty will not, in general, coincide with the policy derived optimally for either of the two models alone.

3.2 <u>*The efficient locus of combinations of losses in the two models*</u>

Before considering how a central bank might address paradigm uncertainty, we derive a tool that will help describing our results. In the simple models developed in Section 2, the state of the economy at any time t is fully described by the state vector $\mathbf{x}_t$, which has four elements: the output gap; inflation; and the current and one period lagged real money stock. Given the backward looking nature of the models we consider, issues of commitment (which give rise to policy rules involving additional constraints and thus greater complexity than functions of the state variable) do not arise in this context. Therefore, we restrict ourselves to linear policy rules which are a function of the four elements of the state vector (i.e. $i_t = g\,\mathbf{x}_t$, where g is a vector of policy rule coefficients).

For each policy rule g we can derive the resulting loss in the OGM and the P* model, which we label L_{OG} and L_{P*} respectively. Figure 4 plots pairs of losses from the two models for a variety of policy rules in the L_{OG} / L_{P*} space. The line AB shown in Figure 5 surrounds the set of attainable (L_{OG}, L_{P*}) combinations and thus represents the locus of points which, for a feasible value of L_{OG}, minimise the value of L_{P*} and vice versa, i.e.

$$L_{OG} = m\,(L_{P*}) \text{ where } m\,(L_{P*}) = L_{OG}(h) = min_g\,\{L_{OG}(g)\} \text{ subject to } L_{P*}(h) = L_{P*} \qquad (14)$$

This locus can be derived for a given parameterization of the two underlying structural models using numerical methods. (Figures 4 and 5 use the parameter values from Table 1.) For reasons of notational simplicity, we label this locus AB *M*. A monetary policy rule *h* is associated with each point on *M*. As will be developed further below, the locus *M* is a useful expositional tool for further analysis. It exhibits a number of noteworthy features.

First, the shape of the *M* is a function of *all* the parameters introduced in Sections 2 and 3 (and listed in Table 1). While it is natural that the curve depends on the structural parameters of the two models under consideration, *M* is also influenced by central bank preferences (y) (since these enter the calculation of the losses for the two models). In addition, *M* depends on the covariance matrices of the structural shocks in the two models (S_i; i = OG, P*), since certainty equivalence in linear-quadratic models of the type discussed here only holds for optimal policy rules, not for other rules such as those tracing the interior of the locus *M*.

Second, it is straightforward to pin down two points on *M*, namely those which are associated with the optimal rules for each of the two structural models (points A and B in Figure 5). These rules can be derived using standard optimal control techniques as described in Section 2.4. The corresponding loss in the other paradigm can be obtained simply by substitution of the rule into the alternative model. A and B correspond to the points illustrated in the Taylor curves in Figures 3a and 3b. Because of the optimality of the policy rules underlying these two points, the locus *M* is always to the right of the line AC (since $L_{P*}(g) = L_{P*}^{min}, \forall g$) and above the line BD (since $L_{OG}(g) = L_{OG}^{min}, \forall g$).

Third, *M* is convex to the origin. The intuition underlying is simplest to follow close to the rules defining points A and B. These points are associated with the optima for the two underlying models. In a smooth linear model such as those considered here, small perturbations to the policy rule parameters in the vicinity of the optimal rule should produce modest changes in the loss function for the model in which that rule is optimal (since close to a minimum the sensitivity of $L_i(g)$ to small changes in the parameters of the policy rule will be modest). However, small perturbations to this rule will produce larger changes in the loss for the other model (since away from the optimum rule the sensitivity of $L_j(g)$ to policy rule parameter changes will be large). This result is also illustrated by Figure 4.[6]

Fourth, it is straightforward to see that movements in a "north-easterly" (NE) direction in the L_{OG} / L_{P*} space are unambiguously undesirable: they entail an increase in the loss for both of the structural models being entertained by this paper. By the same token, movements in the SW direction are unambiguously desirable: they lower the loss in both models. Therefore, where M is positively sloped, central banks should always favour policy rules that shift along M in a SW direction.

Fifth, where M is negatively sloped (as in the interval AB in Figure 5), an unavoidable trade off exists between losses in the two models. One cannot choose a policy rule that will reduce the loss in one of the models without raising the loss in the other model. In what follows, we discuss various approaches which central banks may use to confront this trade off.

4. Monetary policy rules in the face of paradigm uncertainty

In this section, we consider how central banks might address the paradigm uncertainty that underlies the multiple model framework discussed above. We consider three approaches: first, various schemes that apply weights to the models; second, a method based on the minmax criterion familiar from game theory; and, third, methods relying on the construction of encompassing inflation forecasts which combine analysis from both models. Each of these approaches address the issue of how to arrive at a single monetary policy rule for short-term interest rate decisions, while nevertheless combining the analysis and implications of each of the two models or paradigms being entertained.

4.1 Weighting schemes

a) Weighting the loss functions

A natural starting point to combine the information revealed by analysis of the two structural models would be to assign weights to the two loss functions and optimize over the combined problem. In this context, the central bank's problem can be written as:

$$\textit{minimise} \quad E\ \mathbf{x}'\mathbf{R}\,\mathbf{x} \quad \textit{where}\ \ \mathbf{x} = \begin{bmatrix} \mathbf{x}_{OG} \\ \mathbf{x}_{P*} \end{bmatrix}; \quad \mathbf{R} = \begin{bmatrix} q\mathbf{R}_{OG} & 0 \\ 0 & (1-q)\mathbf{R}_{P*} \end{bmatrix}$$

$$\textit{subject to} \quad \mathbf{x}_{t+1} = \begin{bmatrix} \mathbf{A}_{OG} & 0 \\ 0 & \mathbf{A}_{P*} \end{bmatrix}\mathbf{x}_t + \begin{bmatrix} \mathbf{B}_{OG} & \mathbf{B}_{P*} \end{bmatrix} i_t + \begin{bmatrix} e_{OG} \\ e_{P*} \end{bmatrix}_{t+1} \qquad (15)$$

$$\textit{and} \quad \begin{aligned} i_t &= F\mathbf{x}_t \\ F &= \begin{bmatrix} f & f \end{bmatrix} \end{aligned}$$

Prima facie this problem appears very similar to the standard optimal control problem discussed in Section 2.4. However, the final constraint (F = [f f]) precludes the straightforward adoption of dynamic programming methods. This constraint requires that the policy rule employed by the central bank is identical in the two models, i.e. the central bank must specify how it will respond to developments in the state variables without knowing which of the two models it is confronting. This constraint reduces the number of policy rule parameters to be chosen and thus, in the language used by Söderlind (1998), results in a simple policy rule, rather than the fully optimal rule. (Note that the terminology here is somewhat misleading. The constraint is a fundamental part of the problem posed by paradigm uncertainty, not a simplification introduced to reduce computation time or for presentational ease.)

As Söderlind (1998) demonstrates, problem (15) can be solved using numerical techniques. Note that the solution will not, in general, exhibit certainty equivalence. In particular, as the presence of correlation between the fundamental shocks in the two models could reduce or increase the trade-off between losses in the two models, any correlation between ε_{OG} and ε_{P*} will affect the chosen policy rule.

As *q* varies from zero to one, the solution to problem (15) traces out the efficient *M* locus. By construction, the solution to this problem will therefore yield an efficient outcome, in the sense that the resulting policy rule will lie on *M*. In other words, every point on *M* in the range AB in Figure 5 is supported by a solution to problem (15) for some value of *q* between zero and unity.

The intuition behind this result is straightforward. If one is minimizing a weighted average of the two loss functions, one will always chose the lowest attainable loss in model *i* for given loss in model *j* and vice versa. Therefore, problem (15) can be restated in a form that replicates the definition of the efficient locus *M*, as captured in expression (14).

If *q* is interpreted as a Bayesian prior probability for the OGM (chosen exogenously and a priori by policy makers), this approach provides a criterion for identifying a unique optimal monetary policy rule which will be efficient (at least in the specific, well-defined sense used in this paper).

One can restate problem (15) in simpler terms as:

$$\text{minimize}_g\, K = [q\, L_{OG}(g) + (1-q)\, L_{P*}(g)] \qquad (16)$$

subject to the constraints imposed by the two underlying structural models. Using the terminology introduced by Svensson, this statement of the problem can be viewed as defining a “target rule” for monetary policy in the face of paradigm uncertainty. In other words, the monetary policy rule is defined implicitly as a solution to a well-defined economic problem. Note, however, that in this context the resulting rule will depend on the Bayesian prior probability or weight (*q*), which is determined outside the model.

As shown in Figure 6, minimization of a weighted average of the two loss functions involves finding a point of tangency between M and a (negatively sloping) line with gradient $(q - 1)/q$. A natural benchmark rule would set $q = 0.5$ (representing a uniform Bayesian prior over the two paradigms). Assuming that the shocks in the two paradigms are uncorrelated (as is the case for the parameterization in Table 1), the procedure implied by (15) then minimizes the mean of the two loss functions. In Figure 6, this results in the selection of the monetary policy rule associated with point E, the point of tangency between M and a negatively sloped 45° line ($L_{OG} = -L_{P*}$). The resulting mean loss is $K_{q=0.5}$.

Table 4 reports the optimal Bayesian rule based on a weighting of the loss functions with $q = 0.5$ and compares it with the optimal rules for each of the two underlying structural models.

b) <u>Weighting the two structural models</u>

An alternative approach to a Bayesian weighting scheme would assign a subjective prior distribution over parameter values. To maintain the essence of the multiple paradigm approach, we assign weights to each of the two models rather than to individual parameters. q represents the weight on the OGM and $(1-q)$ the weight on the P* model. More complex distributional assumptions for the underlying parameters are possible. However, if the weighting does not apply to each model as a whole but rather to the individual parameters in the unrestricted version (11), then giving a structural interpretation to the system may prove difficult. More importantly, in a monetary policy setting it would imply that policy-makers have to agree on a reference model and then consider the uncertainty surrounding some of its parameters or features. This might be very difficult, especially in a collegial decision-making body.

Weighting models rather than individual parameters can also be interpreted as making strong assumptions about the *joint* prior distribution of the individual parameter values. In particular, the implicit joint distribution associated with this approach implies that concerns about the controllability of the system under some configurations of parameter values do not arise. (This is one of the issues identified by the existing literature as a cause of the attenuation result deriving from analysis of so-called Brainard (1967) uncertainty.) In this paper, we assume that we are either in one or the other of the structural models. Since price developments are controllable using monetary policy in both models, issues of controllability do not arise.

The resulting weighted average model is then amenable to the same analysis as discussed in Section 2.4, whereby optimal control techniques allow an optimal policy rule to be derived. The problem facing the central bank can be written as:

$$\textit{minimize} \quad L(\mathbf{x}) \tag{17}$$

$$\textit{subject to} \quad \mathbf{x}_{t+1} = (q\mathbf{A}_{OG} + (1-q)\mathbf{A}_{P*})\,\mathbf{x}_t + (q\mathbf{B}_{OG} + (1-q)\mathbf{B}_{P*})\,\mathbf{i}_t + (q\mathrm{e}_{OG,t+1} + (1-q)\,\mathrm{e}_{P*,t+1})$$

This problem can be solved using conventional linear quadratic optimal control techniques (since the problem merely amends the transition equation in a linear manner). The results for the parameterization shown in Table 1, assuming a uniform prior distribution (q = 0.5, giving the two paradigms equal weights), are shown in Table 5.

By comparing Table 4 with Table 5, it is evident that the coefficients of the policy rule which minimizes a weighted average of the loss functions are greater than those obtained when the models themselves are weighted. This runs counter to the well-known Brainard (1967) result that parameter uncertainty leads to less aggressive interest rate responses to economic developments. The intuition behind this result follows from the previous discussion of the different impact of paradigm and parameter uncertainty on controllability. Since controllability is not a concern in the context of paradigm uncertainty (as we have described it), there is no reason to attenuate policy responses that would potentially be destabilizing if the system were not controllable.

c) Weighting the optimal rules from the two paradigms

Another weighting scheme involves constructing a weighted average of the optimal policy rules for each of the two underlying structural models, i.e.

$$i_t \quad = \quad g\,\mathbf{x}_t \quad = \quad q\, f_{\mathrm{OG}}\,\mathbf{x}_t \quad + \quad (1\text{-}q)\, f_{\mathrm{P*}}\,\mathbf{x}_t \qquad (18)$$

Note that this rule is *ad hoc* in the sense that it is not derived from a well-specified optimization problem facing the central bank.

If the optimal rules are linearly independent, it is possible to obtain any interest rate simply by choosing an appropriate value of q. Therefore, if q is time varying, any policy rule can be replicated using this approach. However, such a procedure suffers from one of the following two shortcomings. On the one hand, if the weight q is chosen in an entirely ad hoc fashion, the procedure has little claim to be rule based. On the other hand, if a well-specified procedure for determining the choice of weights exists, this procedure needs to be explained. In practice, this would imply a more complicated rule than suggested by the simple formulation above.

To overcome these problems, it may prove to be more useful to consider rules that adopt a fixed weight q at the outset, where q is again seen as a Bayesian prior probability for the OGM. Figure 6 illustrates the range of outcomes achievable in the L_{OG} / $L_{\mathrm{P*}}$ space as q varies between zero (corresponding to point A, where the central bank adopts the optimal rule for the P* model) and one (corresponding to point B, where the optimal rule for the OGM is adopted).

By construction, the locus of L_{OG} and $L_{\mathrm{P*}}$ combinations must lie on or inside M. As is reflected in Figure 7, in general this locus will be strictly inside the efficient locus M, i.e., the locus derived from rules like (18) is not as convex as M. In economic terms, this implies that rules of the form suggested by equation (18) are not efficient, in the sense that alternative rules exist which allow a lower loss to be obtained in *both* the P* model and the OGM. However, under the model parameterizations adopted in this paper, the potential gain is relatively small.

The intuition behind this result is as follows. Since we are entertaining two models, a weighted average of the optimal rules implied by the two models spans a two-dimensional space. However, given the dimension of the state vector (4×1), one needs to span a four-dimensional space if an efficient rule (i.e. one which corresponds to a point on *M*) is, in general, to be obtained. This is not possible with a formulation such as (18). Expressed simply, by choosing the weight *q* appropriately, one can replicate two coefficients of an efficient rule, but in general this will not guarantee that the coefficients on the other two elements of the state vector are appropriate.

This result has a significant policy implication. It demonstrates that there is a return to integrated analysis of the two underlying structural models. If a rule like (18) were able to obtain efficient outcomes on *M*, this would suggest that analysis of the two models could be undertaken independently. Having derived the optimal rule for each model, all that is required is an appropriate weight to be chosen. The preceding analysis demonstrates that, in general, the outcome resulting from this approach can be improved upon in *both* of the underlying structural models if analysis using the two models is cross-checked and evaluated in an integrated manner. There is a return to using such integrated analysis as input into a single monetary policy rule encompassing both paradigms, which is associated with a point on *M*. This result develops the intuition introduced in Section 3: once paradigm uncertainty exists, deriving the optimal rules for each paradigm is, in general, not sufficient to provide the information required for policy makers to address paradigm uncertainty in an efficient manner.

Table 6 reports the performance of a rule derived from expression (18) with the weight $q = 0.5$.

It should be noted that the rules derived using the approach based on weighting the two structural models (i.e. expression (17)) trace out the same locus in the L_{OG} / L_{P*} space as those based on a weighting of the two optimal rules for the models (i.e. expression (18)). In other words, these two approaches allow the same set of (L_{OG}, L_{P*}) combinations to be attained. However, the same weight will yield different points on this locus, as shown in Figure 7.

Two points follow. First, care is required in setting the weights assigned to each of the two paradigms when a Bayesian approach is being followed. The same weight will give different results depending on the approach adopted to integrating the analysis from each of the two underlying structural models. The weights therefore need to be interpreted with care.

Second, both the weighting model and weighting rule approaches to arriving at a single interest rate rule are inefficient in the sense that any rule derived using these methods can be strictly dominated by another feasible rule which achieves a lower loss in *both* models, which itself lies on the efficient locus M and is associated with the minimization of a weighted average of the two loss functions.

4.2 The minmax rule

Minimizing the average loss in the two structural models, while by construction optimal on this criterion, exposes the central bank to larger losses in one model than the other. In selecting monetary policy rules, central banks may entertain an alternative criterion, which permits a greater mean loss but risks a smaller maximum loss. In other words, a central bank may attempt to insure itself against bad outcomes (the worse case scenario), paying a premium for this insurance in the form of a higher average loss.

The simplest characterization of such an approach (and one widely discussed in the literature) is the adoption of a minmax criterion for selecting among policy rules, i.e.

$$minimize_g\ L_i(g) = \{max_i [L_{OG}(g), L_{P*}(g);\ i = \mathrm{OG}, \mathrm{P}^*]\} \quad (19)$$

In our two-model framework, it is useful to distinguish between two cases in the derivation of the minmax rule characterized by (19). These two cases are distinguished on the basis of condition (20):

$$\exists\ i \neq j \ \ \textit{such that}\ L_j(f_i) < L_i(f_i) \qquad i, j = \mathrm{OG}, \mathrm{P}^* \quad (20)$$

where f_k is the optimal rule for model k.

First, consider the situation where condition (20) does *not* hold. When substituted in model j, the optimal rule from model i yields a loss greater than the minimum loss in model i. As shown in Figure 8, this corresponds to a situation where M is negatively sloped as it crosses the 45° line.

In this situation, the minmax criterion will select the rule that minimizes the mean loss in the two models, subject to the constraint that the losses are equal, i.e.

$$minimize_g\ \ 0.5\,[L_{OG}(g) + L_{P*}(g)]\ \textit{subject to}\ \ L_{OG}(g) = L_{P*}(g) \quad (21)$$

Reasons for this conclusion can be drawn from Figure 8. First, the minmax rule must lie on M. Otherwise it would be possible to simultaneously reduce the loss in both models, something unambiguously desirable on the minmax (or any other plausible) selection criterion. Second, if M is negatively sloped when it crosses the 45° line, moving along M away from the 45° line in either direction only reduces the loss in one model at the expense of raising the loss in the other model. Such an outcome would be undesirable on the minmax criterion, which is concerned only with the maximum loss. Therefore, the minmax rule is associated with the point F in Figure 8, where the locus *M* and the line $L_{OG}(g) = L_{P*}(g)$ intersect.

Table 7 reports the minmax rule derived in this way using the parameteraizations of the two models reported in Table 1.

Second, consider the case where the optimal rule in one model produces a smaller loss when it is substituted into the other model, i.e. condition (20) holds. In this situation, the minmax criterion will select the optimal rule for model *i*. The reason is straightforward. By construction, the optimal rule for model *i* will yield the minimum loss for that model. If that minimum loss is nevertheless greater than the loss obtained when the rule is used in the other model *j*, this minimized loss is the maximum for the optimal rule over the two models. It therefore meets the minmax criterion. This situation is illustrated in Figure 9, which has the distinguishing feature that the locus *M* is positively sloped at the point it cross the positively sloped 45° line ($L_{OG} = L_{P*}$).

(Figure 9 is based on the same parameters as reported in Table 1, with the long-run interest rate semi-elasticity of money demand (γ) lowered from 0.25 to 0.1. Having a lower interest rate elasticity implies that the interest rate changes to dampen the inflationary implications of monetary shocks in the P* model must be greater and thus more destabilizing to output volatility, i.e. the trade off between volatilities is exacerbated. In this situation, losses in the P* model are greater than those in the OGM and, in consequence, condition (20) holds.)

When condition (20) holds, a central bank which has adopted the minmax criterion behaves *as if* it were only concerned about one of the two structural models being entertained. In other words, the central bank's conduct of monetary policy *appears* to ignore the multiple paradigm framework that underlies the analysis presented in this paper, even though its behaviour is firmly rooted in that analysis. Moreover, at least to the extent that the higher values of the loss function arise from larger variances of the estimated shocks, the central bank will tend to focus on that model which – although it cannot be rejected on the basis of the available data – fits the data less well.

A number of features of the minmax criterion are noteworthy.

First, as parameter values and central bank preferences change, the character of the minmax monetary policy rule can change quite dramatically (in a non-linear and potentially non-monotonic manner). For example, small changes in central bank preferences may be sufficient to lead to a shift from a situation where condition (20) holds to one where it fails. In this case, a central bank that was behaving as if only one of the monetary policy paradigms were relevant (i.e. adopting the fully optimal rule for one of the two models) could start behaving in a manner that explicitly recognizes the multiple paradigm framework (i.e. adopting a rule that is optimal in neither model). In practice, this can mean important changes in the monetary policy rule. For example, a central bank behaving as if only the OGM were relevant would not accord monetary developments *any* role in its interest rate decisions. Yet a small change in preferences or model parameters that triggered the failure of condition (20) to hold would result in the adoption of a rule where money played a potentially important role.

Second, since the minmax rule lies on *M*, it is possible to choose a value of *q* in the Bayesian weighting procedure implied by expression (16) that will replicate the minmax rule. Therefore, it is possible to present the minmax criterion as a weighting-based approach, where the weight is not an exogenous Bayesian prior probability but rather a weight determined endogenously by the parameters of the central bank problem (such that the tangency between *M* and the line with slope $(q\text{-}1)/q$ occurs at $L_{OG}(g) = L_{P*}(g)$).

4.3 Forecast-based schemes

A third approach to combining the analysis undertaken within each of the two paradigms into a single interest rate decision relies on the construction of forecasts, in particular forecasts of inflation. Statistical theory suggests that any two forecasts can always been combined to obtain a single forecast. If monetary policy is formulated on the basis of a single forecast constructed by combining the forecasts from each of the two underlying structural models, this provides a straightforward approach to address paradigm uncertainty.

The logic underlying forecast-based approaches to paradigm uncertainty stems from the extensive literature on inflation targeting. This literature typically places an inflation forecast at the center of monetary policymaking and views it as a summary (or even sufficient) statistic for policy decisions (Batini and Haldane, 1998). However, the label "inflation targeting" has been applied to a wide variety of different operational schemes for monetary policy decision-making. Therefore, when comparing forecast-based schemes for confronting paradigm uncertainty with the Bayesian and minmax approaches discussed above, one should clearly delineate the approach being used.

In this section, three different forecast-based approaches are identified. In contrast to the approach adopted by Levin, et al. (2001), we view the forecast used to define the rule as a vehicle for combining the information and analysis from the two paradigms. Levin, et al. (2001) investigate the performance of monetary policy rules that include inflation forecasts as arguments. In assessing these rules, they use model-consistent forecasts, i.e., the forecast used to derive the desired interest rate is that constructed using the model in which the performance of the rule is being evaluated. In contrast, in our exercise, we use the forecasts derived from each of the underlying paradigms or models to construct a *single* central bank inflation forecast. It is this combined forecast (denoted F_t to distinguish it from the model consistent forecast E_t) that is used to derive the interest rate. The resulting interest rate rule is then evaluated in each of the two paradigms (we discuss the weighting scheme below).

a) *Conditional instrument rules*

Under the simplest form of forecast-based monetary policy rules, interest rates respond to deviations of an inflation forecast from the inflation objective, where the inflation forecast is constructed on the basis of an unchanged short-term interest rate (e.g. Batini and Haldane, 1998). This rule has the attraction of presentational simplicity: when inflation is expected to be higher than desired, short-term interest rates should be raised (and vice versa).

In the context of paradigm uncertainty, this rule would have the form:

$$i_t \quad = \quad k_C \ \{ \ \mathrm{F}_t \ (\Delta p_{t+k} \,|\, \{i_j\}_{j=t\,..\,t+k} = i_{t-1}) \quad - \ \Delta p^* \} \qquad (22)$$

where: $\mathrm{F}_t \ (\Delta x_{t+k})$ is the forecast at time t of variable x at the horizon $t+k$; Δp^* is the inflation objective, which (consistent with expression (12)) is normalized to zero; and, k is the sensitivity to which short-term interest rates respond to deviations of inflation forecasts from the this objective.

b) *Conditional target rules*

An alternative approach to forecast-based monetary policy rules implies that interest rates are set at a level that – if maintained over the forecast horizon – results in a forecast of inflation at the objective level at the chosen forecast horizon.

In the context of paradigm uncertainty, this rule would satisfy the following condition.

$$\textit{choose } i_t \textit{ such that } \mathrm{F}_t \ (\Delta p_{t+k} \,|\, \{i_j\}_{j=t\,..\,t+k}) = i_t) \quad = \quad \Delta p^* \qquad (23)$$

We label this approach a conditional target rule since, first, it relates to a conditional forecast of inflation (i.e. one that does not embody the implications of the rule itself for future interest rates); and, second, the interest rate is defined implicitly rather than as a reaction to deviations of the forecast from the target level.

c) *Unconditional instrument rules*

Finally, we consider forecast-based rules where short-term interest rates respond to unconditional inflation forecasts, i.e. forecasts of inflation which are constructed so as to be consistent with the implications of the rule itself. Rules of this type are similar to those investigated by Levin, et al. (2001). Note that the approach discussed here differs from that employed elsewhere in the literature since the inflation forecast used to determine the path of future interest rates is not the model's best forecast of inflation (i.e. the mathematical expectation of inflation) but rather a weighted average of the forecasts produced by the two models being considered.

In the context of paradigm uncertainty, this rule would be expressed as:

$$i_t \quad = \quad k_U \ \{ \ \mathrm{F}_t \ (\Delta p_{t+k}) \quad - \ \Delta p^* \} \qquad (24)$$

d) Performance of forecast-based rules in the two paradigm framework

Tables 8 through 13 show the performance of the forecast-based rules in the two paradigm framework. For each of the approaches discussed above, two procedures are investigated: first, an (exogenous) weight of 0.5 (corresponding to a uniform Bayesian prior) is used to construct the overall forecast to which short-term interest rates respond (results shown in Tables 9, 11 and 13); second, the weight on the two underlying forecasts is chosen so as to minimize either the average loss across the two models or the maximum loss in either model (Tables 8, 10 and 12),[7] where the losses in the two models are again characterized by the conventional central bank loss function (12).

Where the possibility exists, the parameters of the rule are varied so as to minimize the average loss. The forecast horizon from which the rule dictates that policy decisions feedback is analysed for all rules. The responsiveness of interest rate decisions to deviations of the inflation forecast from target is varied for the two instrument rule procedures. In the tables, the best performing rule (measured using the average loss across the two paradigms) is shown in bold.

A number of conclusions can be drawn from this exercise.

Tables 8 through 13 illustrate a number of results familiar from the extensive literature on inflation targeting. For example, consistent with Batini and Nelson (2000), by lengthening the feedback horizon of the forecast the volatility of inflation and output can be reduced (at least over some range). Consistent with Levin, et al. (2001), as the feedback horizon is lengthened for unconditional forecast-based rules, the system becomes more prone to instability and multiplicity of equilibria. More generally, apparently more sophisticated rules (e.g. the adoption of an unconditional rather than conditional approach) do not necessarily improve performance. Thus many of the attractions and shortcomings of forecast-based rules that have already been widely illustrated in the literature carry over to this two paradigm framework.

More importantly, the forecast-based rules (irrespective of their precise formulation) are not efficient (in the sense used in this paper). In other words, the outcomes obtained using such rules do not lie on the efficient M locus. Alternative rules exist which can improve upon the outcomes obtained using the forecast-based rules in both models under consideration.

This result follows from two inter-related effects, which are hard to disentangle in practice. First, the literature on inflation targeting has amply demonstrated that simple forecast-based rules of the type discussed here are, in general, sub-optimal. The inefficiency of such rules in the two paradigm framework thus reflects, in part, their general shortcomings.

Second (and specific to the framework analyzed in this paper), constructing a single encompassing forecast by weighting the forecasts from each of the two underlying paradigms is not an efficient mechanism for analyzing, combining or responding to the information required for monetary policy decisions. In some circumstances, there may be little difference in the appropriate monetary policy response across the two paradigms and thus little need to moderate policy actions desirable in one paradigm for fear of causing large disruption in the other paradigm. In other circumstances (e.g., following a monetary shock, which is entirely benign in one paradigm but inflationary in the other), the appropriate response of monetary policy may be quite different in the two models. The efficient approach discussed above – which involves distinct analysis of *each* of the underlying models and cross-checking of the implications of that analysis in *all* of the models – can reveal the trade-offs inherent in this situation.

5. Conclusions

Monetary policy makers face considerable and varied uncertainty. When taking monetary policy decisions, policy makers need to be aware of this uncertainty and factor its effects into their interest rate choices.

In this paper, we investigate the implications of paradigm uncertainty for monetary policy making. In particular, we consider – albeit in the context of very simple macroeconomic models – the implications of uncertainty concerning the role of monetary aggregates in the transmission mechanism of monetary policy.

In the face of paradigm uncertainty, a fundamental problem facing monetary policy makers is how to combine information revealed – and analysis conducted – in the context of *each* paradigm into a *single* monetary policy rule that "performs well" across the various paradigms considered. After all, the central bank can only have one policy, regardless of the number of paradigms or models being entertained. Since the paradigms are, in practice, represented by different (sets of) analytical economic models, the issue facing policy makers becomes one of combining analyses from different models into a desired level of short-term interest rates.

This paper considers three solutions to this problem: weighting-based schemes; forecast-based schemes; and, a minmax approach. Consistent with the existing monetary policy literature, each of these approaches is characterized as a monetary policy rule. The results of the paper point to the following conclusions.

First, forecast-based monetary policy rules perform quite poorly even in the very simple two paradigm framework introduced in this paper. In particular, they do not yield efficient outcomes, in the sense that alternative rules exist which would reduce the loss (a weighted average of inflation and output volatility) in both of the paradigms being considered. Second, schemes which weight either the individual models or the optimal rules for each of the individual models also do not yield efficient outcomes (in the same sense). The explanation of these results stems from the following. Policy makers need to consider the implications of policy decisions in each of the two paradigms when coming to an interest rate decision. This requires a framework for analysis with two features. It must keep the paradigms sufficiently *distinct* for an independent assessment of the performance of policy choices in each paradigm to be made (thereby ruling out procedures focused on the evolution of a single, encompassing central bank inflation forecast). Yet, at the same time, the framework must be sufficiently *integrated* for the implications of any specific policy choice in all of the paradigms or models to be considered (thereby ruling out schemes that mechanically weight the two models or the optimal rule for each of the two models).

The third result follows naturally. Efficient rules in the face of paradigm uncertainty are derived from procedures that maintain the distinctiveness of the two paradigms and yet integrate analysis of the losses that rules give in each of the paradigms. Within our framework the resulting rule implies that the interest rate should respond also to monetary developments.

This paper identifies a "set of rules" that meet this criterion and are thus efficient in the sense that no alternative rule exists that can simultaneously lower the loss obtained in *both* of the paradigms being entertained. Policy makers should select a rule within this "set of rules" according to their willingness to specify priors on the likelihood of the two paradigms. If they do specify weights, a weighted average of the loss function for the alternative paradigms can be minimised. Such a procedure allows for a set of rules corresponding to the efficient set of rules. We can label this approach the "Bayesian policy maker". By contrast, if the policy maker is unable or unwilling (or if it proves difficult within a collegial decision-making body) to specify probability distributions, he/she can adopt a minmax procedure. This allows for selecting a specific rule from the efficient set, one that avoids potentially large losses in one of the paradigms at the expense of a somewhat higher (ex ante) average loss across the two paradigms. We have also shown that this latter approach can be formulated in terms of endogenous weights.

The paper therefore suggests the following approach to the design of monetary policy in the face of paradigm uncertainty. Incoming data should be analysed in the context of different models of the economy, each representative of a plausible paradigm of the monetary policy transmission mechanism. For the same set of incoming data, each model will have different implications for the appropriate setting of interest rates. Policy makers should evaluate the implications of one paradigm's interest rate in other paradigms. In taking the final monetary policy decision, policy makers need to evaluate, on the one hand, the magnitude of the losses associated with the policy choice in each of the paradigms against, on the other hand, the plausibility (in case of a Bayesian policy-maker) or the worst case scenario (in case of a minmax policy maker) entailed by each of the paradigms that the policy framework entertains.

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Figure 1a *Impulse responses for the output gap model (OGM)*

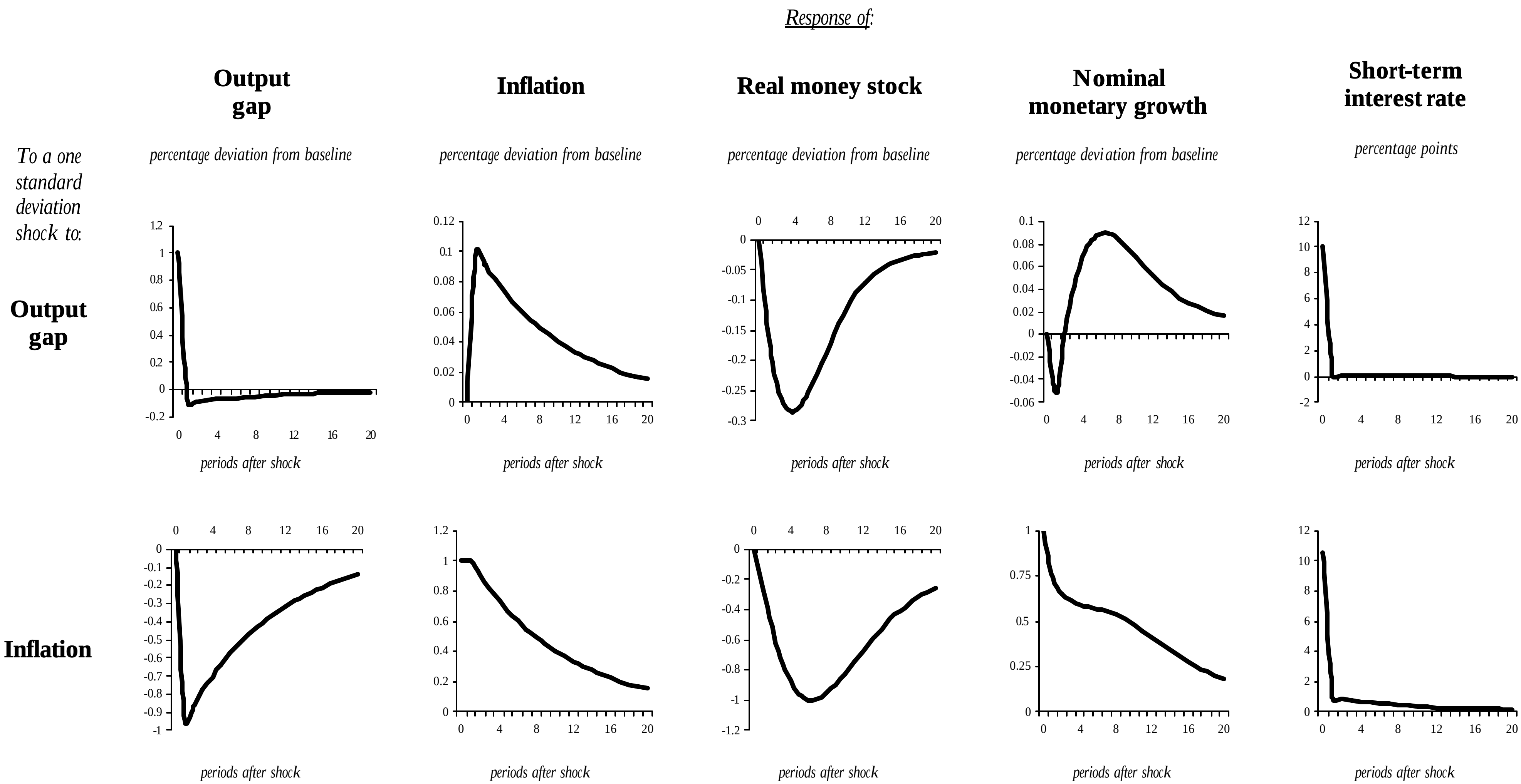

Figure 1a *Impulse responses for the OGM (continued)*

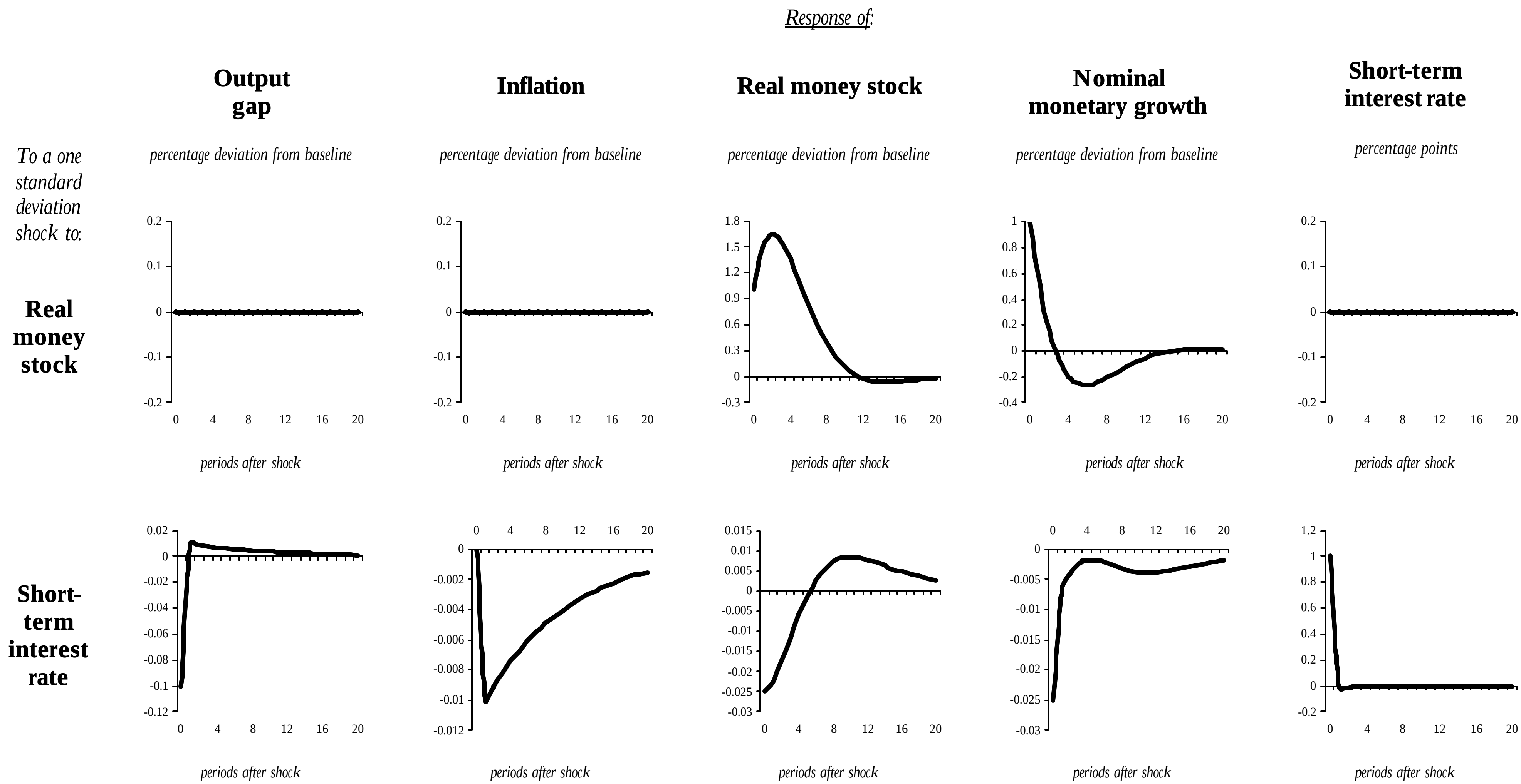

Figure 1b *Impulse responses for the P-star model (P*)*

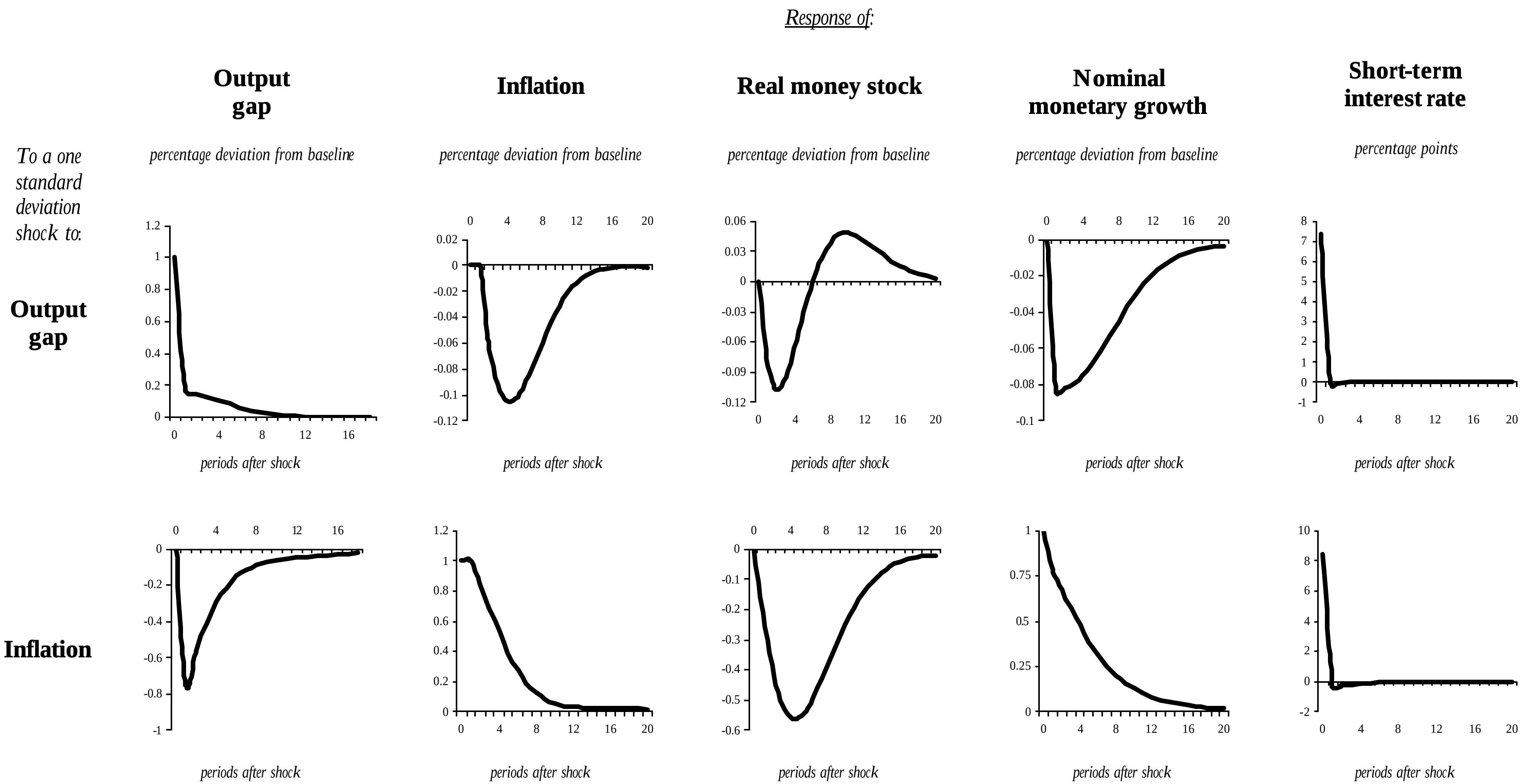

Figure 1b *Impulse responses for the P* model (continued)*

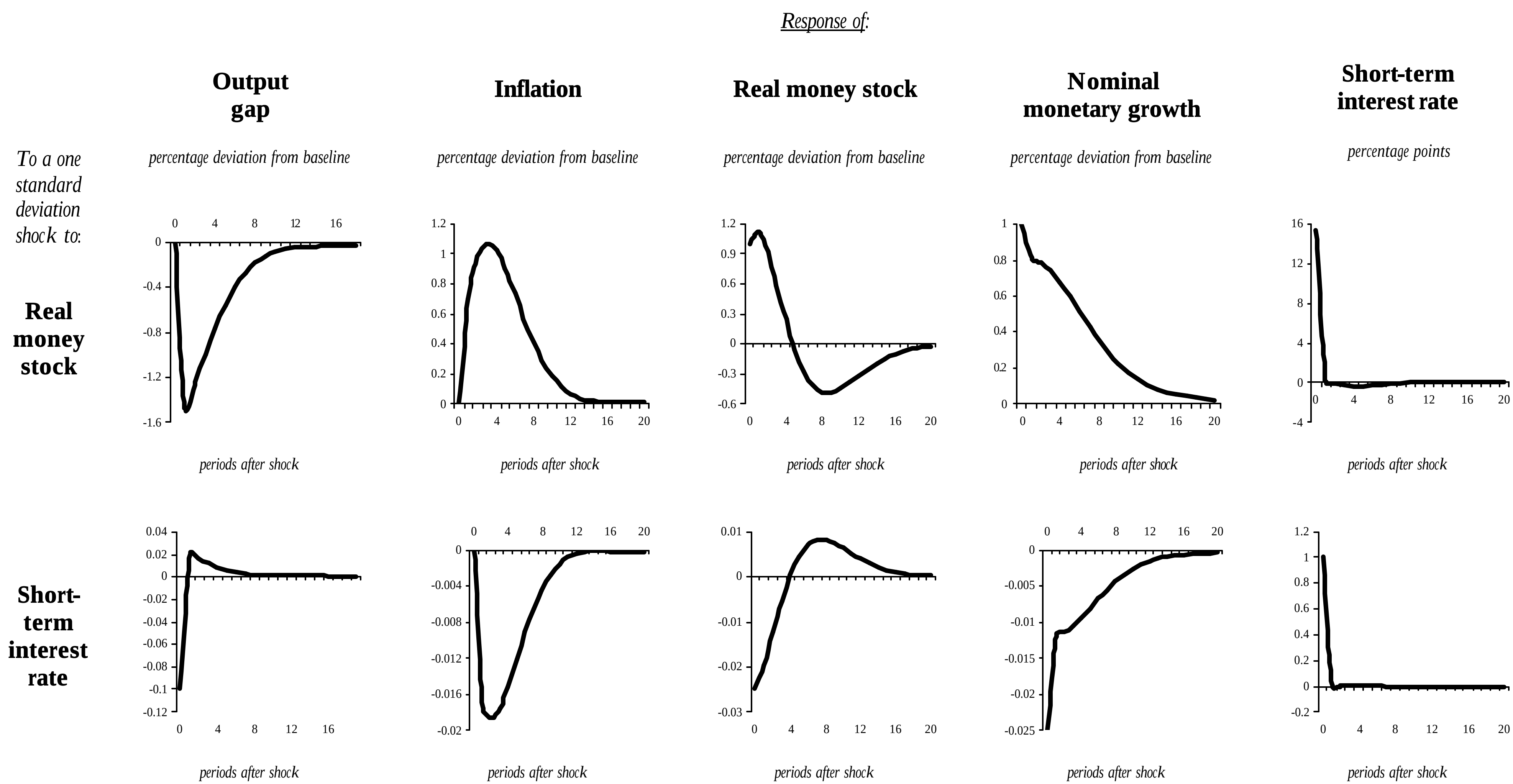

Figure 2a: *Taylor curve for the output gap model (OGM)*

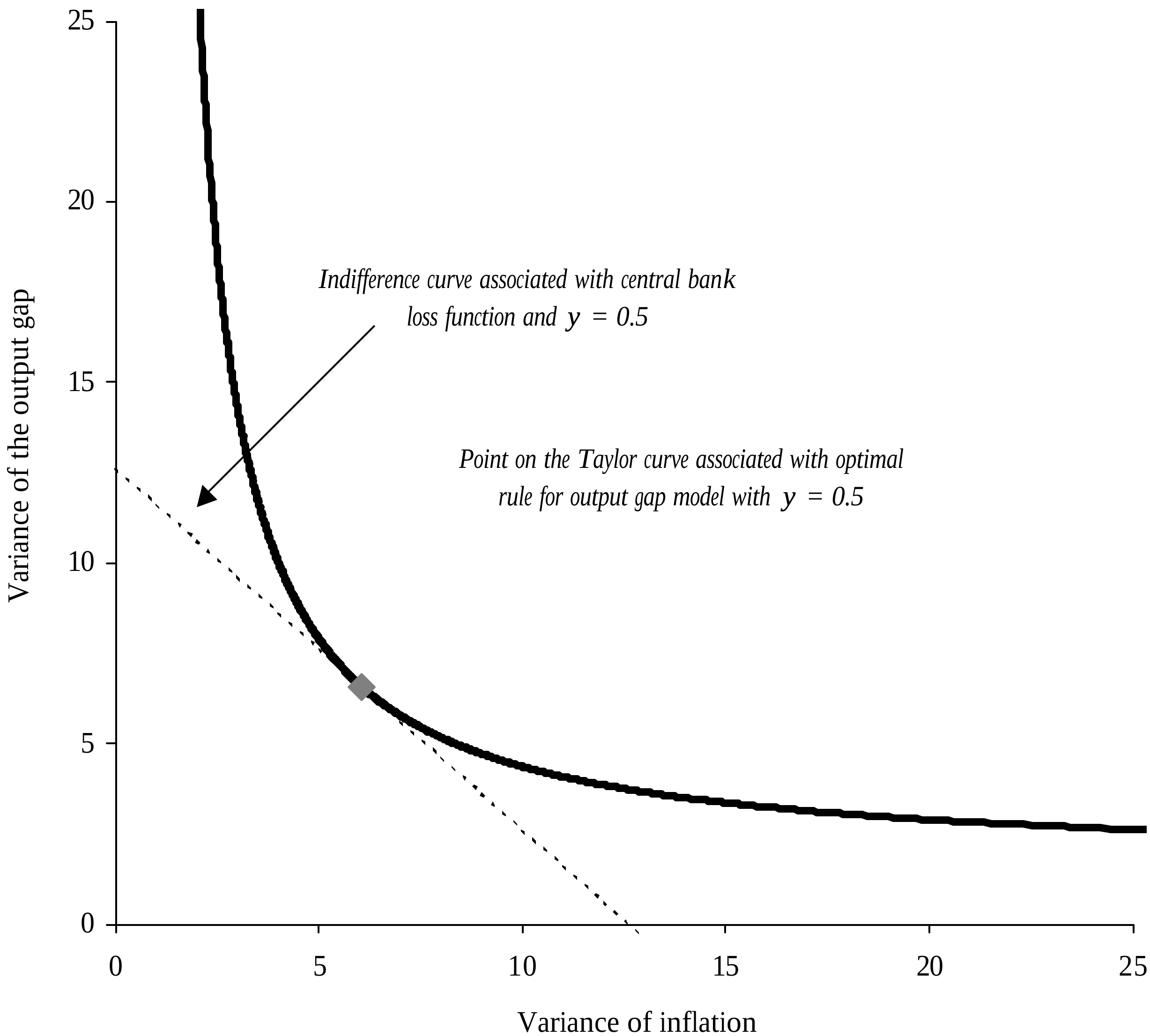

Figure 2b: *Taylor curve for the P star model (P*)*

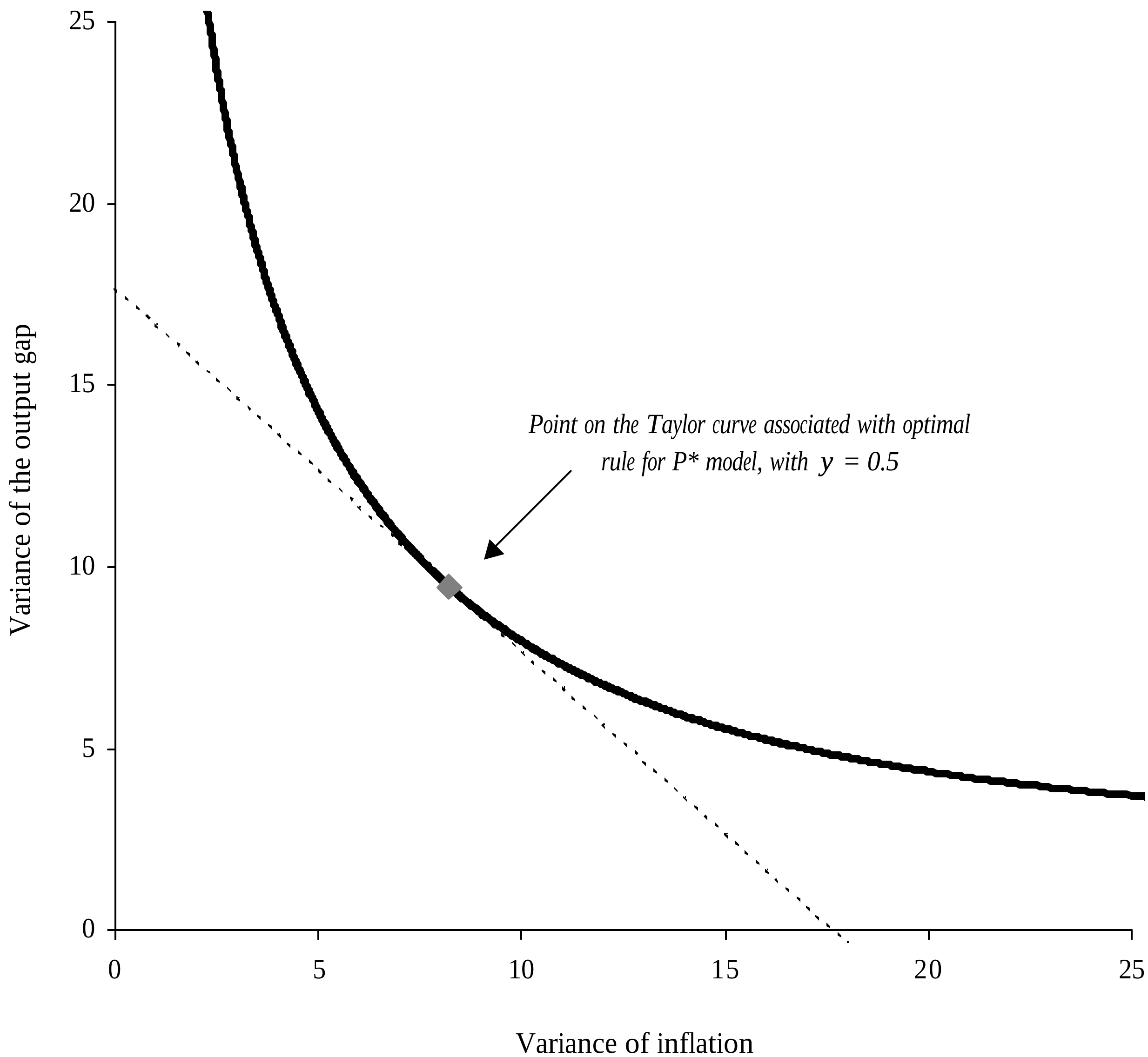

Figure 2c: *Comparison of Taylor curves*

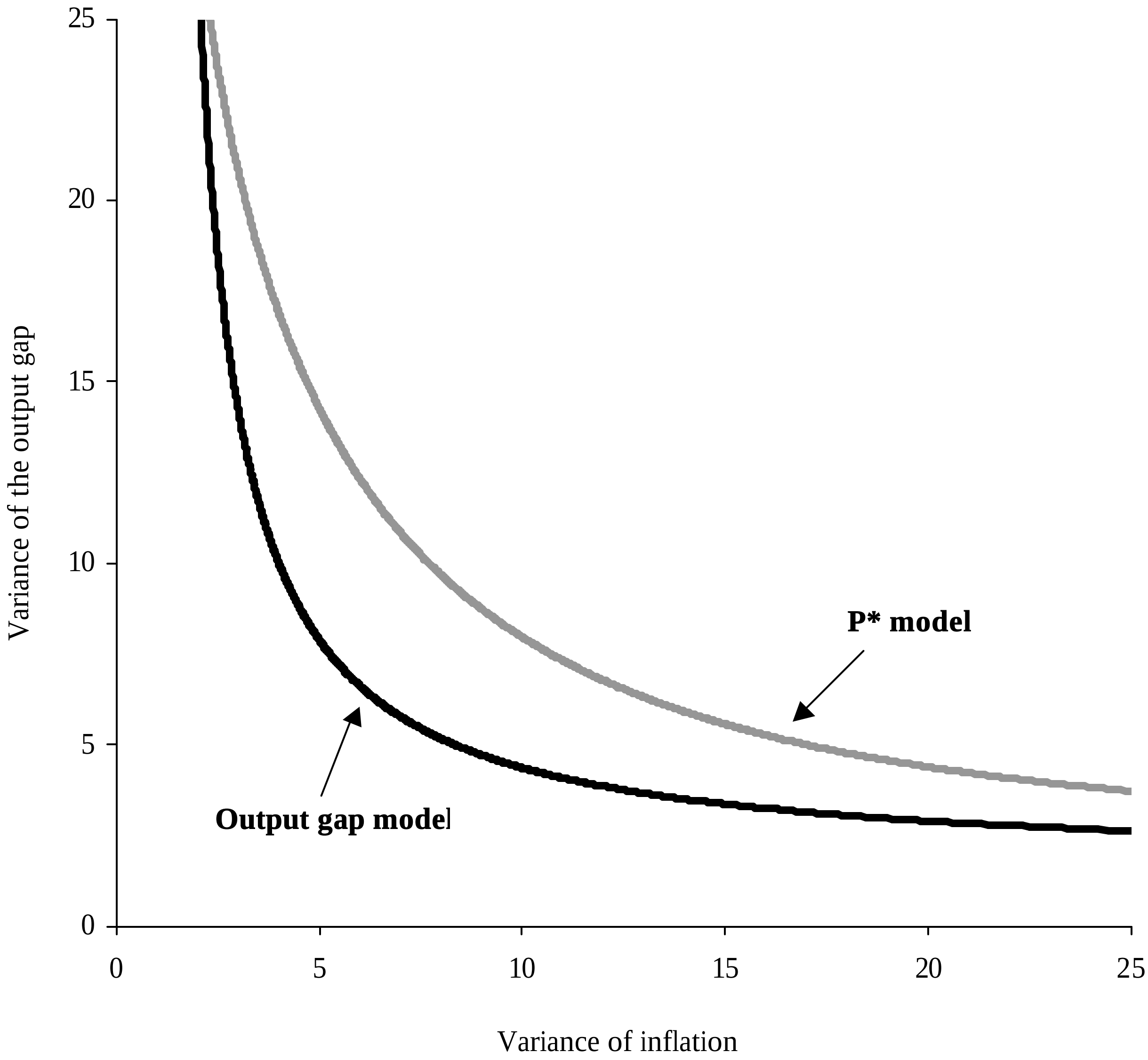

Figure 3a: *Performance of optimal P* rule in the OGM*

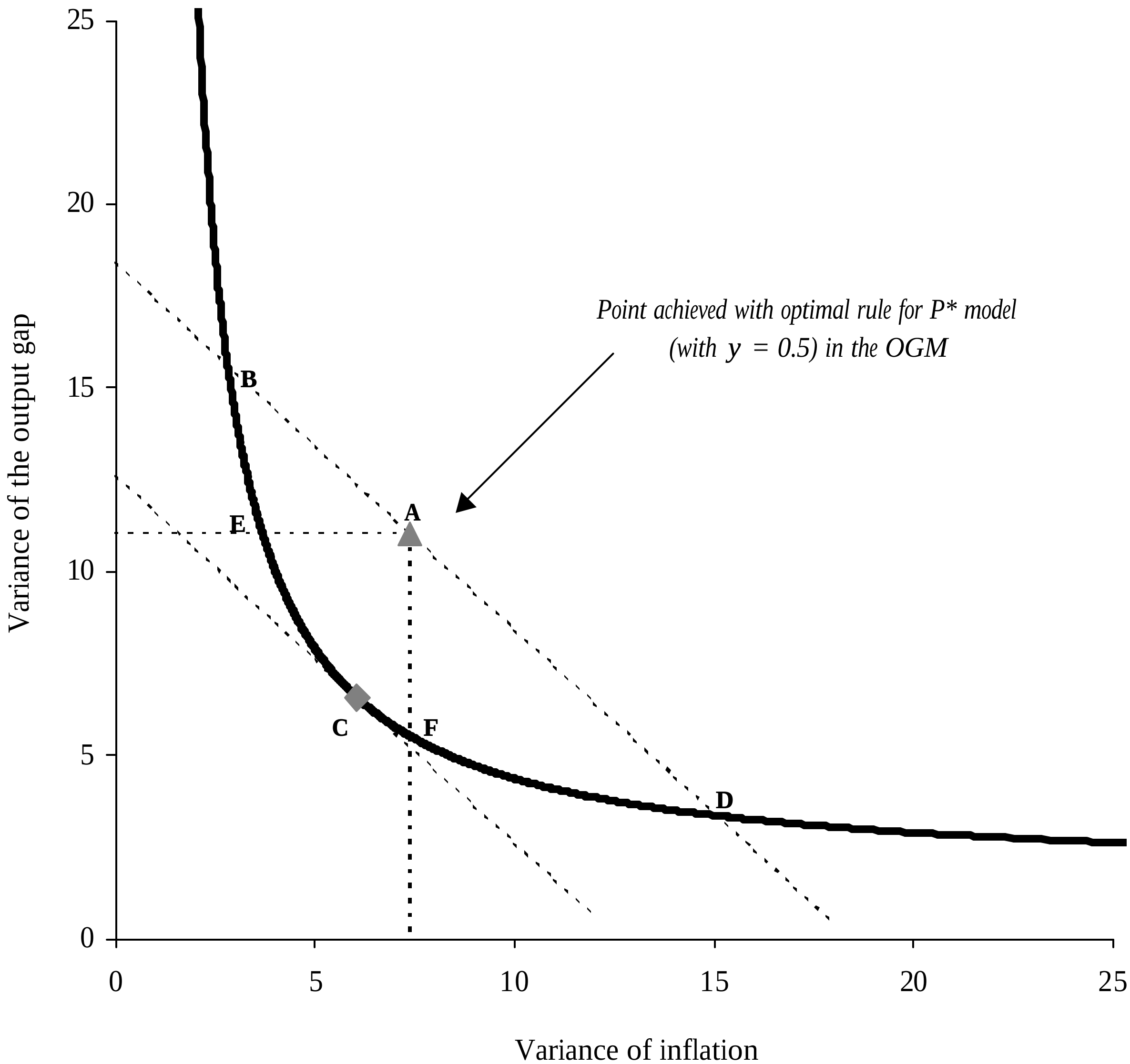

Figure 3b: *Performance of optimal OGM rule in the P* model*

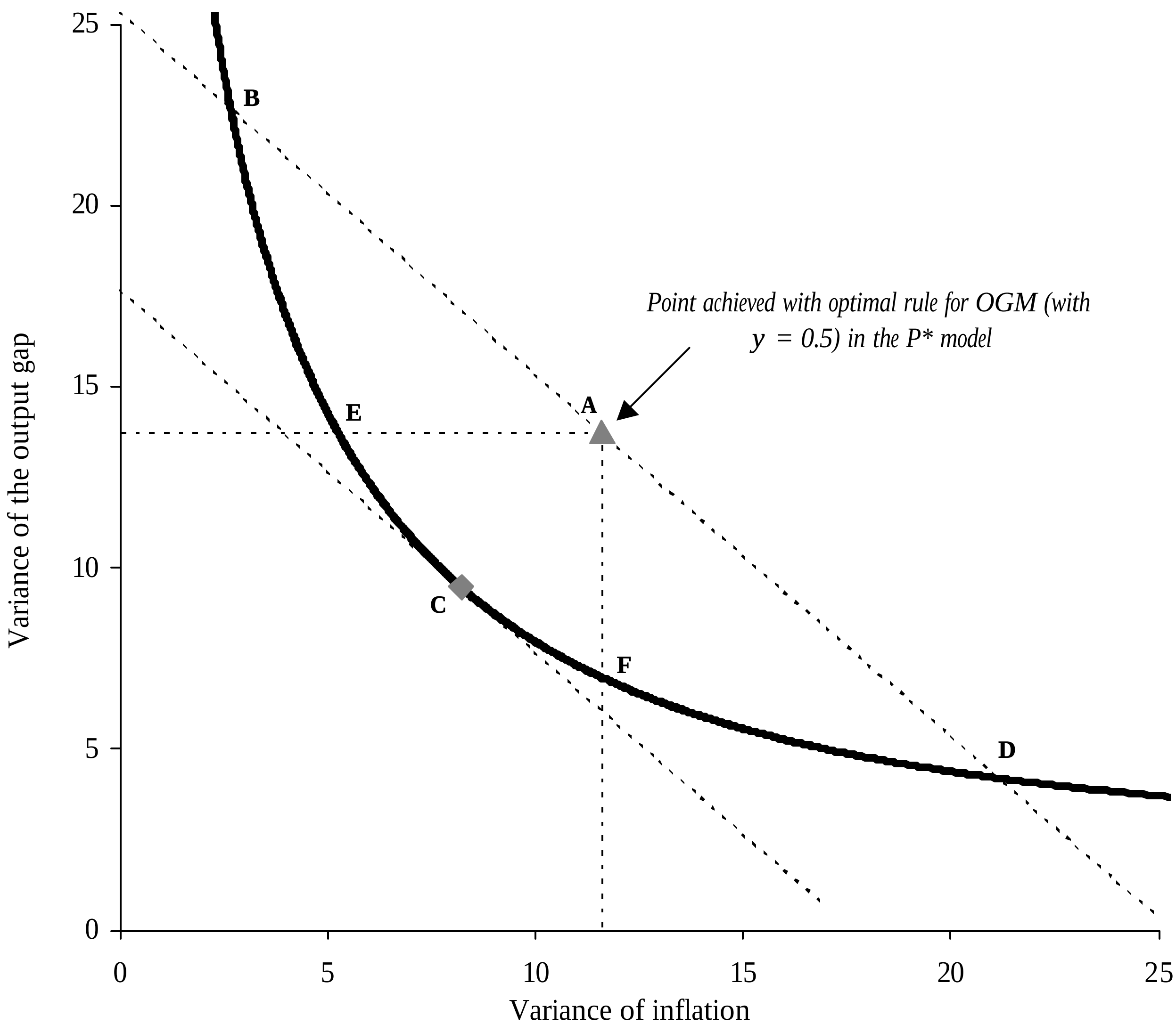

Figure 4: *Locus of efficient L_{OG} and L_{P*} combinations*

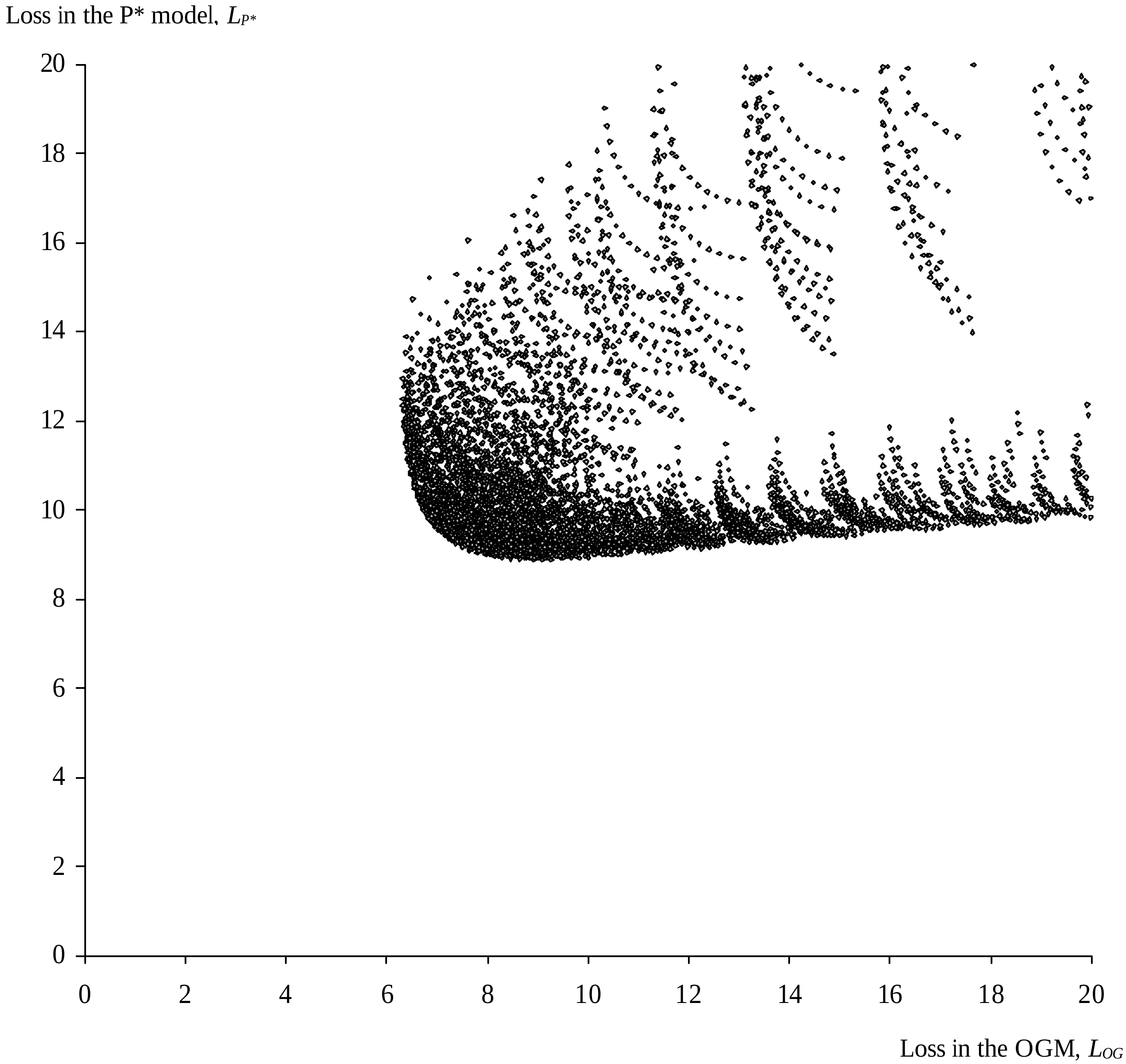

Figure 5: *Locus of efficient L_{OG} and L_{P*} combinations, M*

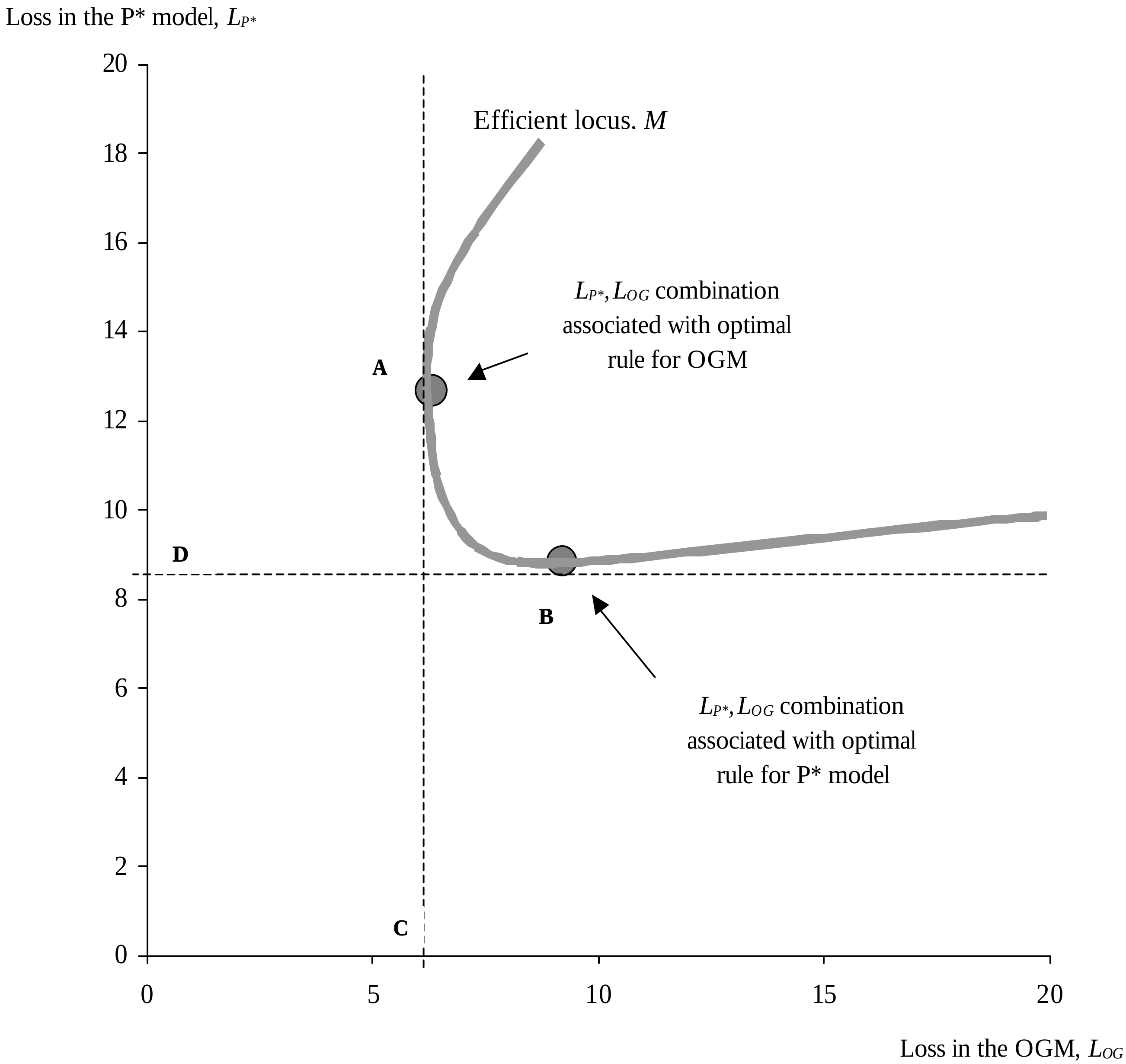

Figure 6: *The Bayesian rule*

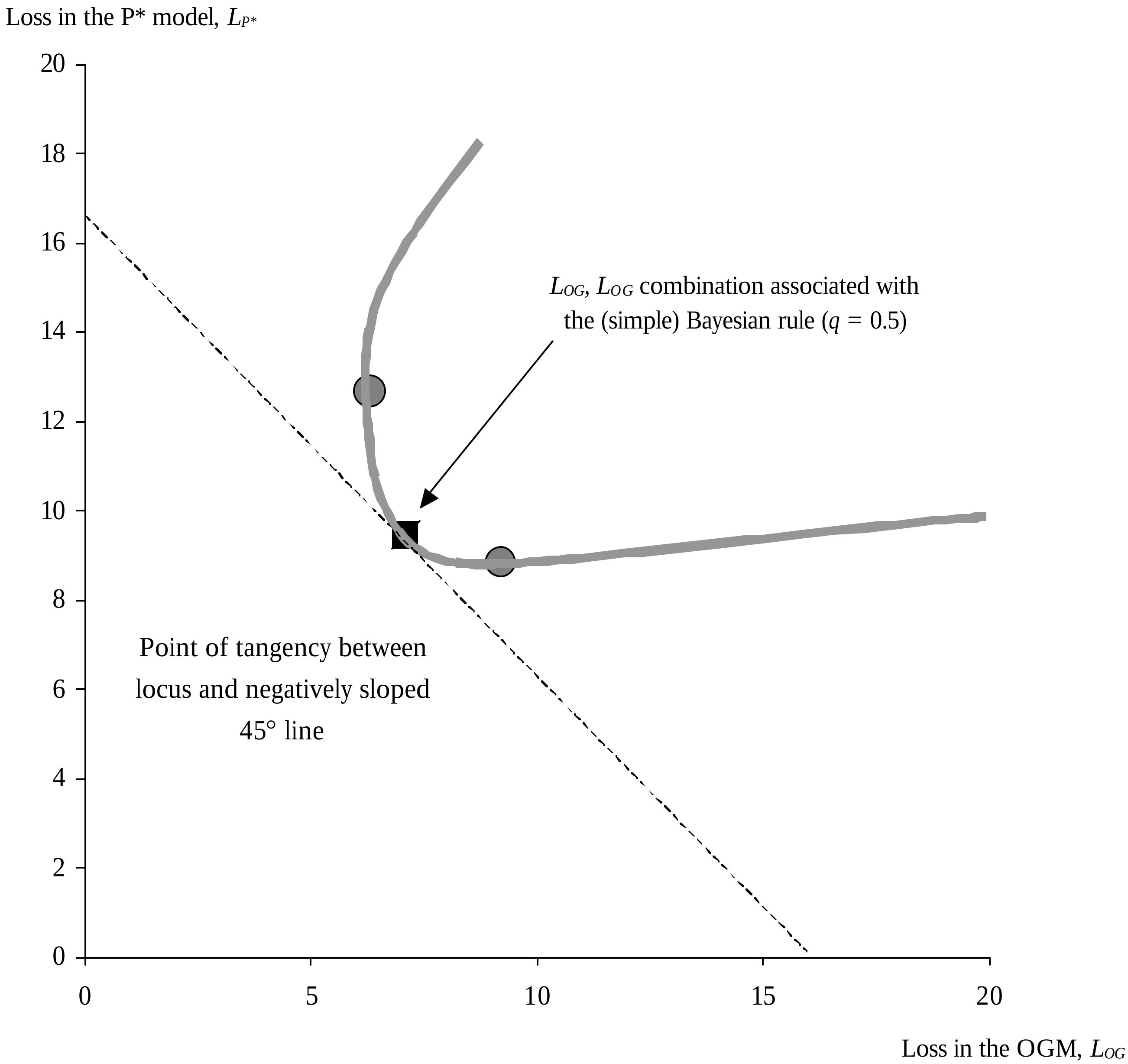

Figure 7: *L_{OG} and L_{P*} combinations associated with weighted models and weighted optimal rules*

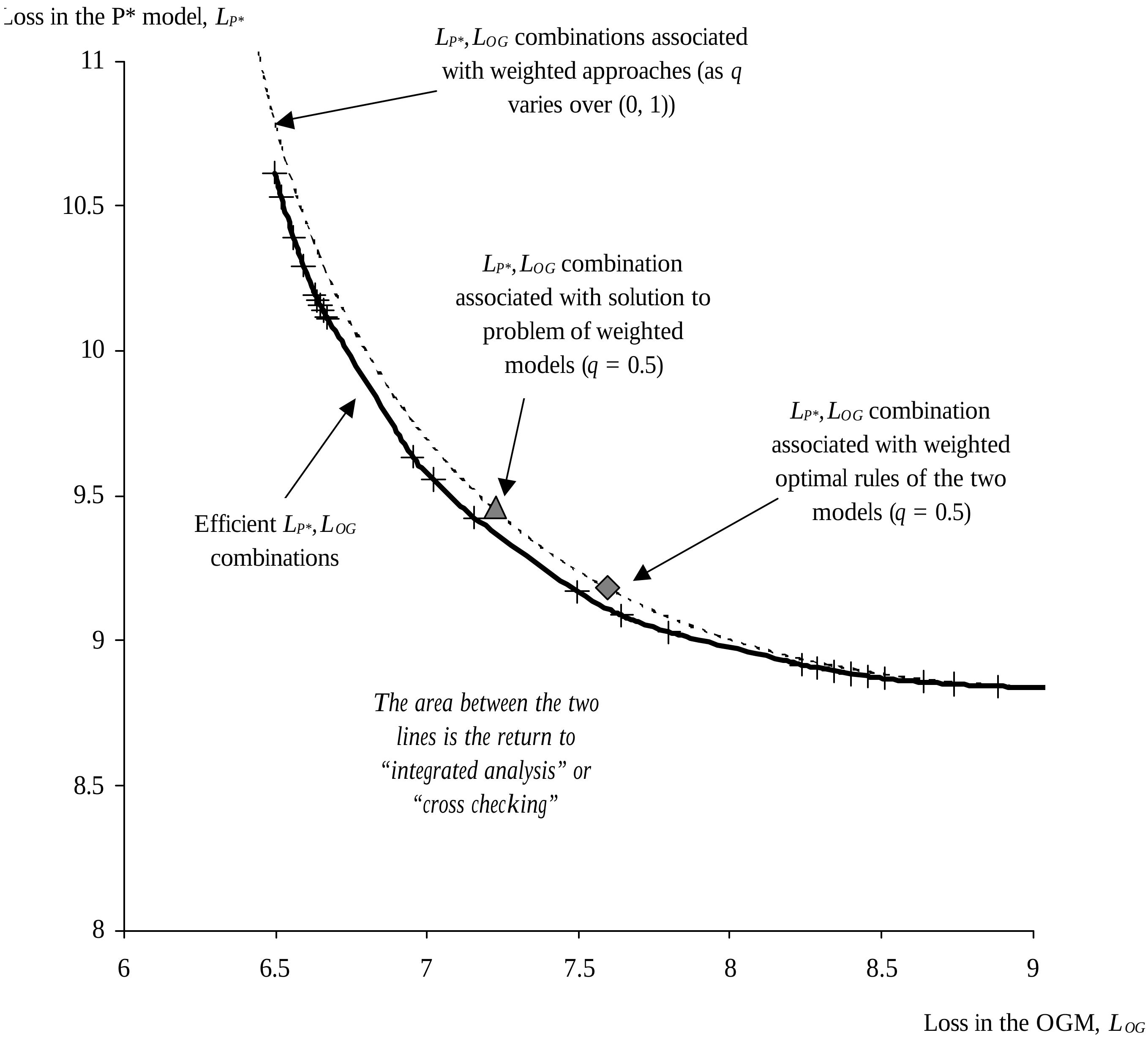

Figure 8: *Minmax rule*

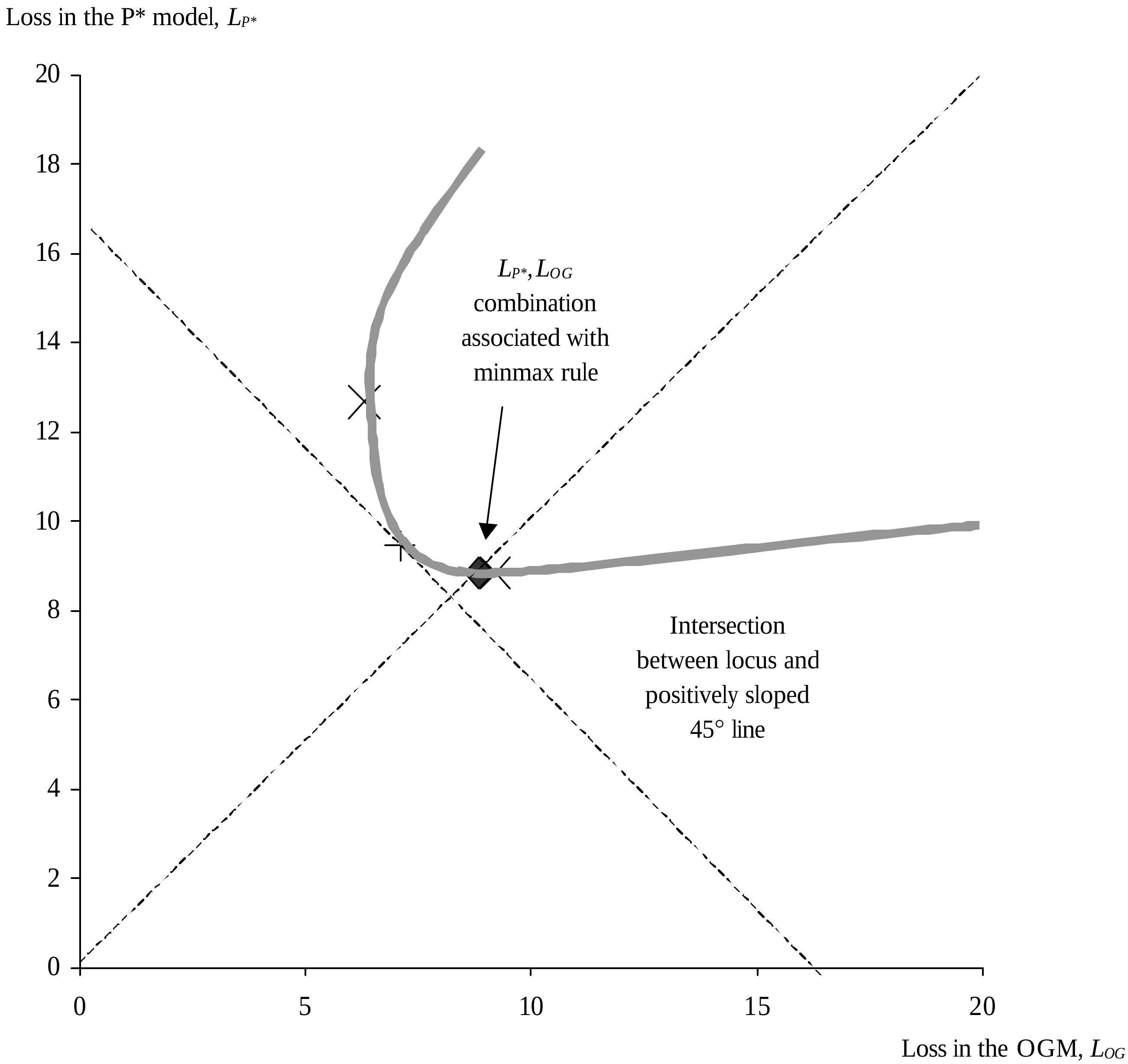

Figure 9: *Alternative case for the minmax rule (recalibrate g = 0.1)*

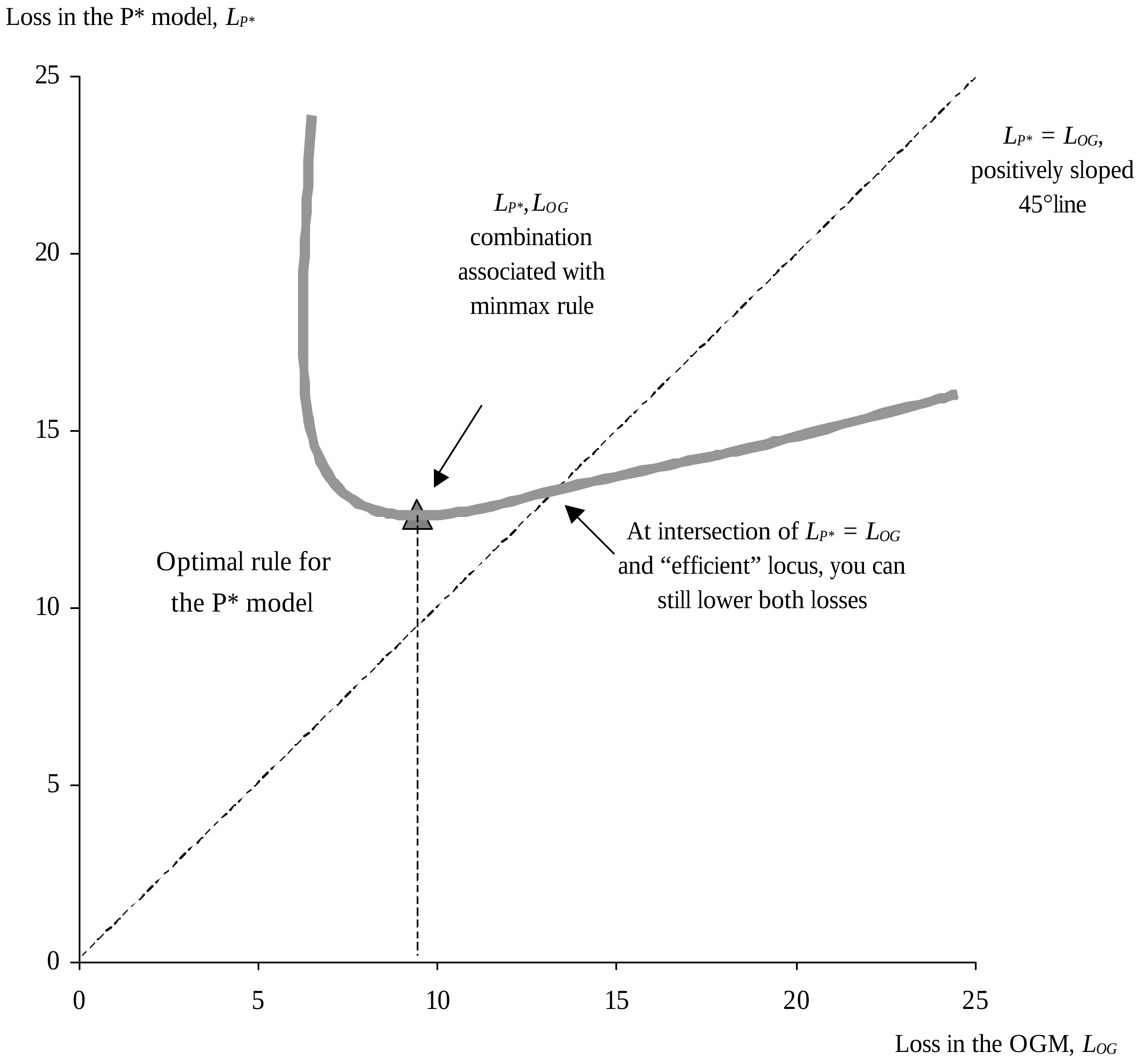

Table 1: *Calibrated values for the model parameters*

Parameter	Calibrated value	Economic interpretation
l	0.9	Output persistence.
d	0.1	Real interest rate elasticity of aggregate demand.
b	0.1	Sensitivity of inflation to the output gap.
f	0.6	Persistence of real monetary growth.
u	0.1	Error correction coefficient in money demand equation.
γ	0.25	Long-run interest rate elasticity of money demand.
ω	0.5	Weight on lagged inflation in P* inflation equation.
μ	0.2	Error correction coefficient in P* inflation equation
$S_{OG} = S_{P*}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	Covariance matrix of the structural economic (demand, supply and money demand) shocks. (For simplicity, a diagonal matrix with unit variances is assumed for both models.)

Table 2: *Coefficients and performance of optimal rules with* $y = 0.5$

	Output gap model	**P star model**
Coefficient in optimal rule on:		
$(y - y^*)_t$	10.051	7.358
Δp_t	10.512	8.472
$(m - p)_t$	0	15.386
$(m - p)_{t-1}$	0	-12.118
Output variance	6.044	8.205
Inflation variance	6.574	9.468
Implied loss, L	6.309	8.836

Table 3: *Performance of optimal rule for model i in model j i,j = OG, P**

	Optimal rule for output gap model in the P* model	**Optimal rule for P* model in the output gap model**
Output variance	11.609	7.381
Inflation variance	13.737	11.040
Implied loss, *L*	12.673	9.210
For reference:	*Performance of optimal rule for P* model in P* model*	*Performance of optimal rule for output gap model in OGM*
Output variance	*8.205*	*6.044*
Inflation variance	*9.468*	*6.574*
Implied loss, L	*8.836*	*6.309*

Table 4: *Coefficients and performance of Bayesan rule weighting loss functions (q = 0.5)*

	Bayesian rule weighting loss functions (q = 0.5)	***OGM optimal rule***	***P* optimal rule***
Coefficient in weighted rule on:			
$(y - y^*)_t$	9.572	*10.051*	*7.358*
Δp_t	9.481	*10.512*	*8.472*
$(m - p)_t$	9.525	*0*	*15.386*
$(m - p)_{t-1}$	-8.190	*0*	*-12.118*
Loss in OGM	7.096	*6.309*	*9.210*
Loss in P* model	9.456	*12.673*	*8.836*
Mean loss	8.276	*9.491*	*9.023*
Maximum loss	9.456	*12.673*	*9.210*

Table 5: *Coefficients and performance of Bayesan rule weighting models (q = 0.5)*

	Bayesian rule weighting models (q = 0.5)	***OGM optimal rule***	***P* optimal rule***
Coefficient in weighed rule on:			
$(y - y^*)_t$	8.541	*10.051*	*7.358*
Δp_t	9.091	*10.512*	*8.472*
$(m - p)_t$	8.344	*0*	*15.386*
$(m - p)_{t-1}$	-6.659	*0*	*-12.118*
Loss in OGM	7.226	*6.309*	*9.210*
Loss in P* model	9.459	*12.673*	*8.836*
Mean loss	8.343	*9.491*	*9.023*
Maximum loss	9.459	*12.673*	*9.210*

Table 6: *Coefficients and performance of Bayesan rule weighting optimal rules (q = 0.5)*

	Bayesian rule weighting rules (q = 0.5)	***OGM optimal rule***	***P* optimal rule***
Coefficient in weighted rule on:			
$(y - y^*)_t$	8.705	*10.051*	*7.358*
Δp_t	9.492	*10.512*	*8.472*
$(m - p)_t$	7.693	*0*	*15.386*
$(m - p)_{t-1}$	-6.059	*0*	*-12.118*
Loss in OGM	7.059	*6.309*	*9.210*
Loss in P* model	9.623	*12.673*	*8.836*
Mean loss	8.341	*9.491*	*9.023*
Maximum loss	9.623	*12.673*	*9.210*

Table 7: *Coefficients and performance of minmax rule*

	Minmax rule	***OGM optimal rule***	***P* optimal rule***
Coefficient in minmax rule on:			
$(y - y^*)_t$	7.554	*10.051*	*7.358*
Δp_t	8.760	*10.512*	*8.472*
$(m - p)_t$	14.987	*0*	*15.386*
$(m - p)_{t-1}$	-12.014	*0*	*-12.118*
Loss in OGM	8.844	*6.309*	*9.210*
Loss in P* model	8.844	*12.673*	*8.836*
Mean loss	8.844	*9.491*	*9.023*
Max loss	8.844	*12.673*	*9.210*

Table 8 *Conditional forecast-based instrument rules*

		Forecast horizon					
		1	**2**	**3**	**4**	**5**	**6**
Mean	*Coefficient on forecast*	2.158	3.205	7.326	6.638	5.238	4.342
	Weight on OGM forecast	0	0	0.021	0.303	0.518	0.697
	Loss in OGM	12.483	10.975	10.300	9.098	8.789	8.866
	Loss in P model*	22.003	16.201	11.805	11.141	12.111	13.661
	Mean loss	17.243	13.588	11.053	**10.119**	10.450	11.263
	Maximum loss	22.003	16.201	11.805	11.141	12.111	13.661
Max	*Coefficient on forecast*	1.858	2.612	8.562	5.828	4.255	3.533
	Weight on OGM forecast	0	0	0	0.063	0.196	0.432
	Loss in OGM	13.918	11.730	10.508	10.410	11.265	11.453
	Loss in P model*	21.497	15.950	11.685	10.410	11.265	12.900
	Mean loss	17.708	13.840	11.097	10.410	11.265	12.176
	Maximum loss	21.497	15.950	11.685	**10.410**	11.265	12.900

Table 9 *Conditional forecast-based instrument rules with weight on forecasts set exogenously (q = 0.5)*

		Forecast horizon					
		1	**2**	**3**	**4**	**5**	**6**
Mean	*Coefficient on forecast*	1.967	2.490	12.451	7.572	5.170	3.703
	Weight on OGM forecast	0.5	0.5	0.5	0.5	0.5	0.5
	Loss in OGM	12.865	10.791	9.127	8.333	8.892	10.625
	Loss in P model*	24.830	20.197	16.239	12.494	12.014	12.941
	Mean loss	18.848	15.494	12.683	**10.414**	10.453	11.783
	Maximum loss	24.830	20.197	16.239	12.494	12.014	12.941
Max	*Coefficient on forecast*	1.729	2.099	14.098	7.909	5.238	3.718
	Weight on OGM forecast	0.5	0.5	0.5	0.5	0.5	0.5
	Loss in OGM	14.321	11.905	9.484	8.391	8.908	10.630
	Loss in P model*	24.303	19.792	16.104	12.476	12.010	12.939
	Mean loss	19.312	15.848	12.794	10.433	10.459	11.785
	Maximum loss	24.303	19.792	16.104	**12.476**	12.010	12.939

Table 10 *Conditional forecast-based target rules (setting rate to put forecast at target)*

		Forecast horizon						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Mean	*Weight on OGM forecast*	*Not feasible since one step ahead inflation forecast is predetermined.*	0.390	0.201	0.204	0.267	0.364	0.467
	Loss in OGM		45.194	13.431	10.077	8.957	8.562	8.500
	Loss in P model*		44.917	15.688	11.411	10.395	10.710	11.580
	Mean loss		45.055	14.560	10.744	9.676	**9.636**	10.040
	Maximum loss		45.194	15.688	11.411	10.395	10.710	11.580
Max	*Weight on OGM forecast*	*Not feasible since one step ahead inflation forecast is predetermined*	0.396	0	0.023	0.077	0.100	0.125
	Loss in OGM		45.056	14.518	10.883	9.840	10.002	10.803
	Loss in P model*		45.058	15.089	10.886	9.838	10.005	10.806
	Mean loss		45.057	14.083	10.885	9.839	10.004	10.805
	Maximum loss		45.058	15.089	10.886	**9.840**	10.005	10.806

Table 11 *Conditional forecast-based target rules (setting rate to put forecast at target) with exogenously chosen weight (q = 0.5)*

		Forecast horizon						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Mean / Max	*Weight on OGM forecast*	*Not feasible since one step ahead inflation forecast is predetermined.*	0.5	0.5	0.5	0.5	0.5	0.5
	Loss in OGM		43.027	12.216	9.058	8.127	8.004	8.336
	Loss in P model*		48.662	19.193	13.890	12.036	11.562	11.763
	Mean loss		45.844	15.704	11.474	10.082	**9.783**	10.050
	Maximum loss		48.662	19.193	13.890	12.036	**11.562**	11.763

Table 12 *Unconditional forecast-based instrument rules*

		Forecast horizon		
		1 *same as conditional since one step ahead inflation forecast is predetermined*	**2**	**3** *longer horizons induce indeterminacy or multiplicity*
Mean	*Coefficient on forecast*	2.158	3.091	~ 77 (induces high interest rate volatility; larger value induce indeterminacy).
	Weight on OGM forecast	0	0	0
	Loss in OGM	12.483	11.19	Asymptote ~ 11.4
	Loss in P model*	22.003	16.461	Asymptote ~ 16.4
	Mean loss	17.243	13.795	Asymptote ~ 13.9
	Maximum loss	22.003	16.461	Asymptote ~ 16.4
Max	*Coefficient on forecast*	1.858	2.532	~ 77 (induces high interest rate volatility; larger value induce indeterminacy).
	Weight on OGM forecast	0	0	0
	Loss in OGM	13.918	11.957	Asymptote ~ 11.4
	Loss in P model*	21.497	16.184	Asymptote ~ 16.4
	Mean loss	17.708	14.070	Asymptote ~ 13.9
	Maximum loss	21.497	16.184	Asymptote ~ 16.4

Table 13 *Unconditional forecast-based instrument rules with exogenously chosen weight (q = 0.5)*

		Forecast horizon		
		1 *same as conditional since one step ahead inflation forecast is predetermined*	**2**	**3** *longer horizons induce indeterminacy or multiplicity*
Mean	*Coefficient on forecast*	1.967	2.450	As large as possible (induces increasing interest rate volatility).
	Weight on OGM forecast	0.5	0.5	0.5
	Loss in OGM	12.865	10.914	Asymptote ~ 12.2
	Loss in P model*	24.830	20.403	Asymptote ~ 18.3
	Mean loss	18.848	15.658	Asymptote ~ 15.2
	Maximum loss	24.830	20.403	Asymptote ~ 18.3
Max	*Coefficient on forecast*	1.729	2.071	As large as possible (induces increasing interest rate volatility).
	Weight on OGM forecast	0.5	0.5	0.5
	Loss in OGM	14.321	12.066	Asymptote ~ 12.2
	Loss in P model*	24.303	19.984	Asymptote ~ 18.3
	Mean loss	19.312	16.025	Asymptote ~ 15.2
	Maximum loss	24.303	19.984	Asymptote ~ 18.3

Notes

1. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank. We are very grateful to Klaus Masuch and Hans-Joachim Klöckers for their helpful comments. The paper was originally prepared for the central bank workshop on monetary policy rules to be held at the ECB in Frankfurt on 11-12 March 2002.
2. The difficulty of rejecting models on an empirical basis will be exacerbated by typical econometric concerns, such as short data samples and multi-colinearity.
3. For an early application of this approach to the design of monetary policy in presence of model uncertainty, see for example Becker, et al. (1986).
4. In the context of such forward-looking, New Keynesian models, the correct measure of inflationary pressure us given by the gap between real wages and the marginal product of labour. Under some conditions, this gap can be re-written as an output gap measure.
5. The passive role of money within this framework is revealed by the block recursive structure of expression (5). In other words, the upper right elements of the matrix $\mathbf{A}_{OG}$ are zero.
6. In fact, for the remainder of the paper, we assume that M is smooth, continuous and convex to the origin (although, since we rely on numerical methods, we do not prove this). Some of the results obtained below rely on these assumptions.
7. Note that this does not imply that the forecast from which policy decisions feedback is necessarily the optimal inflation forecast.