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# Integrating Imputed Interest on the Stock of Equity Provisions into Business Valuation: A Discounted Cash Flow Approach 

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#### Abstract

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#### Abstract

In the last two decades several European countries implemented tax systems that allow for the deduction of imputed equity interest from a company's tax base. This paper integrates the tax benefits resulting from imputed interest on the stock of equity into business valuation. Three alternative discounted cash flow valuation methods are used to this end: the equity method, the Adjusted Present Value (APV) method, and the entity method. Intertemporal differences in risk require the use of various risk-adjusted discount rates in the equity method as well as the APV method. Using the equity method we show that the well-known, market-to-book ratio of the constant growth dividend discount model also holds in our model. When applying the APV method with imputed equity interest, an adjustment is necessary for each business, also for an unlevered company, to account for the tax shield resulting from equity financing. A closed-form solution is presented for the value of this tax benefit. We furthermore derive the weighted average cost of capital (WACC) under imputed interest on the stock of equity and the adjustment of the cost of equity that is necessary to derive the WACC as a weighted average of cost of equity and debt. JEL classification: G24, H25, K34, M40

Keywords: Business Valuation, DCF Methods, Imputed Interest on the Stock of Equity, Cost of Capital, Market-to-Book Ratio, Capital Budgeting


## Introduction

The relative advantage of debt finance over equity finance taking into account the tax consequences of either is common knowledge in finance theory and stems from the deductibility of debt interest payments from the tax base of companies. From a capital structure perspective, this makes debt finance more attractive than equity finance, thereby distorting financing decisions of corporations. Attempting to reduce the distorting effect of taxes on the capital structure choice of companies, the idea of deducting (fictitious) interest on a company's equity in the same way as debt interest was born in the 1980s. Since then, legislators and scientists in several countries have taken up this idea and created tax regimes in which the deductibility of imputed equity interest compensates at least in part for the preferential tax treatment of debt. In light of the potential consequences of the implementation of the new Basel accord on capital adequacy the perception of the need for equity finance has even increased.

Generically, these tax regimes work as follows: in addition to interest on debt, imputed interest expenses on equity are deductible from a company's tax base. To this end, an equity interest rate is fixed for tax purposes by government bodies. The amount resulting from equity interest rate times a company's stock of book equity is then deducted from a company's tax base, just as any other expense. The remainder is subject to the applicable tax rate. In general, however, equity interest does not remain untaxed but is taxed at a lower rate compared to ordinary income. These tax regimes therefore resemble and are sometimes even referred to as "Dual Income Tax Regimes" in which part of the businesses earnings (i.e., the imputed equity interest) are taxed at a reduced rate. A tax regime, where imputed interest on equity is fully excluded from taxation, can be seen as a special case with a zero tax rate for the
imputed interest on equity. Such regimes are denoted as "Allowance for Corporate Equity" (ACE) or "Interest Adjusted Income Tax" (IAIT).

Examples of tax regimes where the imputed interest on equity is subject to a reduced tax rate are the tax systems in several Scandinavian countries. Norway introduced the deductibility of imputed equity interest in the 1990s, followed by Finland. Similar systems, however more complex, are in place in Denmark and Sweden (Soerensen 1998, 2001). Other examples of tax regimes with interest on the stock of equity are the current tax system in parts of Bosnia-Herzegowina (Rose 2004) as well as the former tax systems in Croatia (Rose and Wiswesser 1998), and Italy (Valente 1997, Smith and Valente 1998, Bordignon et al., 2001). For the United Kingdom a corresponding "Allowance for Corporate Equity" has been suggested by the Institute for Fiscal Studies (IFS 1991). Similar proposals have recently also been put forward for changes in the Swiss legislation (Neue Züricher Zeitung of November 2, 2000, Keuschnigg 2004, Keuschnigg and Dietz 2004) as well as for changes in the German legislation (Wagner and Wenger 1999, Arbeitsgemeinschaft Selbständiger Unternehmen 2000, Rose 2003, German Council of Economic Experts 2004). ${ }^{1}$ As a variant, the Austrian tax regime allows for fictitious interest on equity increase to be deducted (IBFD 2001, Doralt and Ruppe 2001). Furthermore, tax regimes with different tax rates for retained and distributed earnings, for example, the former tax regime in Germany or the tax system prevailing in Estonia, can be seen as a special case of such a tax regime that allows for equity interest on the equity increase (by setting the fictitious equity interest rate equal to $100 \%$ ).

[^0]The prime reasoning for the existence of imputed equity interest has been its neutrality with respect to both capital structure and capital budgeting choices of companies. Several studies deal with capital structure issues in the respective tax systems (Boadway and Bruce 1984, Devereux and Freeman 1991, Bogner et al., 2002, Previtero 2003, Bontempi et al., 2004). Previtero and Bontempi find that during the validity of the Italian system that allowed imputed equity interest, the debt ratio of Italian firms had significantly decreased. Fane (1987) investigates the neutrality of a tax regime with imputed interest on equity with respect to a firm's capital budgeting decision. Bond and Devereux (1995) show the robustness of this result in face of bankruptcy risk and wind-up decisions. Panteghini (2001) includes real options in a model with interest rate uncertainty into the neutrality discussion. Löffler and Schneider (2003) demonstrate that a system with an "allowance for corporate equity" is an "investment decision", neutral even if the applicable tax rate is time dependent.

Given the legislation referred to above, the preferential treatment of equity has to be incorporated into existing business valuation models as well. This is especially relevant for businesses with large equity ratios. This paper provides a framework for the valuation of businesses when the tax regime allows for imputed interest on the stock of equity to be considered. In a multi-period, value-driver model under uncertainty (similar to Copeland et al., 2000), we extend common discounted cash flow (DCF)—namely, the equity method, the adjusted present value (APV) method and the entity method-to include the deductibility of imputed interest on the stock of equity. ${ }^{2}$ We thereby derive an expression for the cost of equity of a levered business taking into account equity tax shields as well as a market-to-book ratio endogenous to our model and establish a link to the market-to-book ratio in the Gordon constant

[^1]growth dividend discount model. As for the APV method, a closed-form solution is presented for the value of the tax benefit arising from equity financing. We show that the risk-adjusted discount rates for the tax shields from imputed equity interest are identical to those for debt tax shields. Finally, we derive the weighted average cost of capital (WACC) to be used under the entity method, including the appropriate adjustment to the cost of equity reflecting the value of equity tax shields. Thereby, we generalize the textbook formula for the WACC going back to Modigliani and Miller (1958). We point out that our methodology and the derived cost of capital relations also form the basis for capital budgeting decisions in a tax system with imputed interest on equity.

The structure of the paper is as follows: Section 2 presents the model. Section 3 covers the growth in expected equity in our model. Section 4 deals with the valuation methodology used. Section 5 uses the equity method to calculate the business value in consideration of imputed interest on the stock of equity. Section 6 isolates the contribution of equity tax shields to the business value by means of the APV method. In Section 7 we derive a weighted average cost of capital expression to take into account imputed interest on the stock of equity. Finally, Section 8 concludes the paper.

## Section 2: Model

The object of our study is the following: a company that has an infinite life span and riskless debt outstanding in the form of perpetual bonds with a par value of $D_{t}$ at any time $t .{ }^{3}$ Book equity at time $t$ is symbolized by $E Q_{t}$. The capital structure at book values, denoted by $v=D_{t} / E Q_{t}$, is assumed to be constant over time. ${ }^{4}$

Uncertainty arises from the stochastic development of the book return on investment before interest and taxes in each year $t$ (symbolized by $R O I_{t}$ ), which is calculated as the ratio of the earnings before interest and taxes in year $t, E B I T_{t}$, to the total book capital at the beginning of year $t=$ total capital at the end of year $t-1), T C_{t}$. 1, specifically after payment of business taxes, distribution of earnings and capital increase for the year $t-1$. This model is based on the book return on investment because the imputed interest on equity is based on book equity. The book returns on investment in different years are assumed to be uncorrelated. ${ }^{5}$ The return on investment in year $t$, expected on the basis of the information available at $t-1$, is constant over time and described by the parameter $\overline{R O I}$. It is assumed that in any case the return on investment is sufficiently high to allow the use of all tax shields. ${ }^{6}$ The covariance (on the basis of the information available as of $t-1$ ) between the return

[^2]on investment in year $t$ and the return of the market portfolio in year $t$ is constant over time and symbolized by $\rho$.

At the end of year $t$, the total capital is increased by the product of EBIT in year $t, E B I T_{t}$, and the constant factor $q$. Thus, in states of higher earnings, more capital is put into the business than in states with lower earnings. The parameter $q$, which can be interpreted intuitively as the (net) plowback ratio (net investment rate according to Copeland et al., 2000) is a consequence of the investment program and of the business's resulting financing needs. The increase of total capital is performed by equity or debt with due attention to the constant capital structure requirement, $v=$ $D_{t} / E Q_{t}$. The equity increase is performed by means of either plow-back or issue of new equity, whereas the debt increase is implemented by issuing new perpetual bonds.

In order to make our model applicable to a wider range of countries we do not replicate a tax system of a specific country but model a generic tax system based on the common features encountered in the countries with imputed interest on the stock of equity: earnings are subject to a constant tax at the rate $\tau_{K}$. Debt interest and imputed interest on equity can be deducted from earnings for tax purposes. The imputed interest on equity for year $t$ is calculated by multiplying the book equity at the beginning of year $t, E Q_{t-1}$, by the equity interest rate $r_{e}$, which is assumed to be constant over time. Deductions for imputed interest on equity are subject to preferential tax treatment at the rate $\tau_{S}<\tau_{K}$. Taxes for year $t$ are to be paid at the end of year $t$. Personal taxes are disregarded in this paper (Ross et al., 2002).

## Section 3: Evolution of the expected stock of equity over time

To begin with, let us focus on the growth of expected equity over time within our model: as the increase in the total book capital in year $t$ is $q E B I T_{t}$ from which the $R O I$ definition equals $q T C_{t-1} R O I_{t}$, the relative increase in total capital is $q R O I_{t}$. As the book capital structure is assumed to remain constant, the relative capital increase must be the same for equity and debt, namely $q R O I_{t}$. Therefore:

$$
E Q_{t}=E Q_{t-1}\left(1+q R O I_{t}\right)
$$

The expected value of $E Q_{t}$, conditional on the information available as of time $t-1$, is

$$
E_{t-1}\left[E Q_{t}\right]=E_{t-1}\left[E Q_{t-1}\left(1+q R O I_{t}\right)\right]=E Q_{t-1}(1+q \overline{R O I})
$$

Where $E[\bullet]$ stands for the unconditional expectation and $E_{t}[\bullet]$ denotes the expected value conditional on the information available at time $t$. The unconditional expected value of equity, using the law of iterated expectations, therefore is:

$$
E\left[E Q_{t}\right]=E Q_{0}(1+q \overline{R O I})^{t}
$$

Thus, the expected equity in our model grows at an annual rate of $\bar{g}=q \overline{R O I}$.

## Section 4: The Valuation Methodology of Fama

A valuation model is required for the valuation of the cash flows from the firm. In the following, we will use the model of Fama (1977), which is based on the assumptions of the one-period CAPM derived by Sharpe (1964), Lintner (1965) and Mossin (1966). In addition, Fama generally assumes non-stationary, deterministic risk-free interest rate and market price of risk.

Although several alternative models have been put forward, we opted for the 1977 Fama model because of its compatibility with the one-period CAPM frequently
used in industry and its easy implementation which facilitates the tractability of valuation formulae. ${ }^{7}$

In our model, in addition to the assumptions of Fama, we assume that the riskfree interest rate and the market price of risk are constant over time. Let the risk-free interest rate be denoted by $r_{f}$, and the market price of risk by $\lambda$.

Fama uses the following equation for the value of a cash flow stream $Z_{1}, Z_{2}, \ldots Z_{T}$
(Fama 1977: 19):

$$
P V=\sum_{t=1}^{T} \frac{E\left[Z_{t}\right]}{\prod_{s=1}^{t}[1+k(s, t)]}
$$

Where $k(s, t)$ is the discount rate for the cash flow occurring at time $t, Z_{t}$, for the period from $s-1$ to $s$ (the rate to be applied when discounting from time $s$ to time $s-1$ ).

For the time being, let us assume the following cash flow structure:

$$
Z_{t}=E Q_{t-1}\left(a R O I_{t}+b\right)
$$

## Equation 1

Where $a$ and $b$ are deterministic parameters. As will be shown later in the paper, all cash flows in our model can be represented in this way. For this cash flow structure, the (unconditional) expected cash flow can be derived using the fact that the returns on investment in different years are uncorrelated and applying the expected growth rate as derived in Section 3:

$$
E\left[Z_{t}\right]=E\left[E Q_{t-1}\right]\left(a E\left[R O I_{t}\right]+b\right)=E Q_{0}(1+q \overline{R O I})^{t-1}(a \overline{R O I}+b)
$$

[^3]Within the Fama framework, the following discount rates obtain for the above cash flow structure (see Appendix 1):

$$
\begin{array}{ll}
k(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{\overline{R O I}}}{1+q \overline{\overline{R O I}}}-1 & \text { for } s<t \\
k(s, t)=\left(1+r_{f}\right) \frac{a \overline{R O I}+b}{a \overline{\overline{R O I}}+b}-1 & \text { for } s=t
\end{array}
$$

## Equation 2

With $\overline{\overline{R O I}}=\overline{R O I}-\lambda \rho$. The symbol $\overline{\overline{R O I}}$ can be interpreted as the certainty equivalent of the return on investment, that is, the risk-adjusted expected return on investment. The product $q \overline{\overline{R O I}}$, therefore, is the certainty equivalent of the annual company growth rate (i.e., the risk-adjusted expected growth rate), which will be symbolized by $\overline{\bar{g}}$. Thus, the risk-adjusted discount rate for $s<t$ is obtained using the risk-free interest rate and the ratio of the actually expected growth factor to the riskadjusted expected growth factor. The risk-adjusted discount rate for $s=t$ is calculated using the risk-free interest rate and the ratio of the actually expected cash flow to the risk-adjusted expected cash flow. The reasoning for the difference in $k(s, t)$ between the cases $s=t$ and $s<t$, lies in the cash flow structure of our model $Z_{t}=E Q_{t-1}\left(a R O I_{t}+b\right)$, where risk is resolved differently in the years $s<t$ and $s=t$. Whenever $s<t$, the returns on investment enter into cash flow indirectly through its impact on $E Q_{t-1}$. In the final year $s=t$, however, $R O I_{t}$ affects $Z_{t}$ directly.

Applying the discount rates to the unconditional expectation of the cash flow gives the present value of cash flow $Z_{i}$ :

$$
P V\left(Z_{t}\right)=\frac{E\left[Z_{t}\right]}{\prod_{s=1}^{t}[1+k(s, t)]}=\frac{E Q_{0}(1+q \overline{R O I})^{t-1}(a \overline{R O I}+b)}{\left[\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}\right]^{t-1}\left[\left(1+r_{f}\right) \frac{a \overline{R O I}+b}{a \overline{\overline{R O I}}+b}\right]}=\frac{E Q_{0}(1+q \overline{\overline{R O I}})^{t-1}(a \overline{\overline{R O I}}+b)}{\left(1+r_{f}\right)^{t}}
$$

The numerator on the right-hand term is the certainty equivalent of the cash flow in year $t$. We therefore transform the present value representation with risk-adjusted discount rates on the left-hand side into a certainty equivalent representation on the right-hand side. In the following, we will deal with a growing perpetuity of payments of this structure. With the growth rate of the perpetuity being $q \overline{\overline{R O I}}$ and the first cash flow $E Q_{0}(a \overline{\overline{R O I}}+b)$, this gives a present value of the growing perpetuity $P V$ of:

$$
P V=\frac{E Q_{0}(a \overline{\overline{R O I}}+b)}{r_{f}-q \overline{\overline{R O I}}}
$$

Equation 3
We will show the respective cash flows fit the structure given in Equation 1 for both the equity method and for the APV method. This representation will give the corresponding values for $a$ and $b$. Plugging in the expressions for $a$ and $b$ into Equation 2 and Equation 3 will then give the appropriate discount rates and the present value of the respective cash flow streams.

## Section 5: Equity Method

The equity method will be the first of the three DCF methods covered in this paper. Under the equity method, free cash flows to equity holders are discounted with a rate adjusted to account for financial and business risk as well as imputed interest on equity.

### 5.1 Free Cash Flow to Equity

In a first step, the free cash flow to equity holders that is necessary for valuation is derived. The free cash flow to equity holders, denoted as $F C F^{E}$, is derived from EBIT by deducting debt interest, taxes, capital expenditure, and any increase in working capital (> 0 or $<0$ ) and adding depreciation and debt increase (Damodaran 1992). ${ }^{8}$

EBIT minus debt interest and taxes corresponds to earnings after taxes, denoted as EAT. Furthermore, capital expenditure less depreciation (together: net investments) plus increase in working capital must equal the increase in total capital due to the balance sheet equation. Thus, the free cash flow is earnings after taxes less increase in total capital plus debt increase. As the increase in total capital minus debt increase is, of course, the increase in equity, symbolized by $E I$, the free cash flow to equity holders is simply earnings after taxes minus increase in equity. For year $t$ this gives:

$$
F C F_{t}^{E}=E A T_{t}-E I_{t}
$$

Therefore, computing the free cash flow to equity holders requires the computation of earnings after taxes and equity increase. We begin with earnings after taxes: from the definition of $R O I_{t}, E B I T_{t}$ equals $T C_{t-1} R O I_{t}$. Because debt is risk-free, debt interest amounts to $D_{t-1} r_{f}$. In year $t$, earnings before taxes, $E B T$, are thus:

$$
E B T_{t}=T C_{t-1} R O I_{t}-D_{t-1} r_{f}=E Q_{t-1}\left\lfloor R O I_{t}(1+v)-v r_{f}\right\rfloor
$$

As $D_{t}=v E Q_{t}$. To compute the tax burden, the imputed interest on equity in year $t$ has to be calculated. It amounts to $E Q_{t-1} r_{e}$. The tax base in year $t$, to which $\tau_{K}$ is applied, $T B_{t}$, is the difference between $E B T_{t}$ and the imputed interest on equity in year $t$ :

[^4]$$
T B_{t}=E Q_{t-1}\left\lfloor R O I_{t}(1+v)-v r_{f}\right\rfloor-E Q_{t-1} r_{e}=E Q_{t-1}\left\lfloor R O I_{t}(1+v)-v r_{f}-r_{e}\right\rfloor
$$

The tax liability (including the reduced tax on the imputed interest on equity) in year $t, T A X_{t}$, is thus:

$$
\begin{aligned}
\operatorname{TAX}_{t} & =E Q_{t-1}\left\lfloor R O I_{t}(1+v)-v r_{f}-r_{e}\right\rfloor \tau_{K}+E Q_{t-1} r_{e} \tau_{S}= \\
& =E Q_{t-1}\left[R^{\prime} I_{t}(1+v) \tau_{K}-r_{f} \tau_{K}-r_{e}\left(\tau_{K}-\tau_{S}\right)\right]
\end{aligned}
$$

The expression $\left(\tau_{K}-\tau_{S}\right)$ represents the differential between the reduced tax rate and the ordinary tax rate in the "dual" income tax system. Earnings after taxes in year $t$, $E A T_{t}$, are the difference between earnings before taxes, $E B T_{t}$, and the tax liability in year $t$, TAX $_{i}$ :

$$
\begin{aligned}
E A T_{t} & \left.=E Q_{t-1}\left\lfloor R O I_{t}(1+v)-r_{f}\right\rfloor-E Q_{t-1} \mid R O I_{t}(1+v) \tau_{K}-r_{f} \tau_{K}-r_{e}\left(\tau_{K}-\tau_{S}\right)\right]= \\
& =E Q_{t-1}\left[R O I_{t}(1+v)\left(1-\tau_{K}\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]
\end{aligned}
$$

Having derived the earnings after taxes, as described in Section 3, the increase in equity is $E I_{t}=E Q_{t-1} q R O I_{t}$, so that the free cash flow to equity holders is:

$$
\begin{aligned}
& F C F_{t}^{E}=E A T_{t}-E I_{t}= \\
& \left.=E Q_{t-1} \mid R O I_{t}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)\right]
\end{aligned}
$$

Equation 4
Thus, the free cash flow in year $t$ is composed of the after tax earnings of an equivalent unlevered company which does not use equity tax shields, $E Q_{t-1} R O I_{t}(1+v)\left(1-\tau_{K}\right)$, a reduction for the increase in equity $E Q_{t-1} q R O I_{t}$, which could of course also be negative, the equity tax shields $E Q_{t-1} r_{e}\left(\tau_{K}-\tau_{S}\right)$ and the debt interest charge reduced by the resulting debt tax shields, $E Q_{t-1} v r_{f}\left(1-\tau_{K}\right)$.

### 5.2 Valuation using the Equity Method

The structure of the free cash flow in Equation 4 shows that one can use Equation 1 to Equation 3 by substituting for $a$ and $b$ as follows:

$$
\begin{aligned}
& a=(1+v)\left(1-\tau_{K}\right)-q \\
& b=r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)
\end{aligned}
$$

The discount rate $k_{E}(s, t)$ is therefore:

$$
\begin{aligned}
k_{E}(s, t)=\left(1+r_{f}\right) & \frac{\overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-r_{f}\left(1-\tau_{K}\right)}}{\overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}-1} \quad \text { for } s=t \\
& k_{E}(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}-1 \quad \text { for } s<t
\end{aligned}
$$

Where $k_{E}(s, t)$ is the risk-adjusted discount rate (for the period from $s-1$ to $s$ ) for valuing the free cash flow that occurs in year $t$.

The reason for the difference in $k_{E}(s, t)$ between the cases $s=t$ and $s<t$ lies in the fact that, as explained in Section 0, risk is resolved differently: for all years $s<t$ the return on investment enters into the free cash flow indirectly via $E Q_{t-1,}$, whereas for the last year $(s=t) R O I_{t}$ affects the cash flow directly. Substituting for $a$ and $b$ in Equation 3, the value of equity at an arbitrary time $h$ is:

$$
V_{E Q, h}=\frac{E Q_{h}\left[\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]}{r_{f}-q \overline{\overline{R O I}}}
$$

## Equation 5

Equation 5 can be interpreted as follows: $V_{E Q, h}$ can be derived by applying the present value factor of a growing perpetuity to the certainty equivalent of the first cash flow after time $h$. The certainty equivalent of the first cash flow after time $h$ is $\left.\left.E Q_{h} \overline{\overline{\operatorname{ROI}}[ }[1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]$, the growth rate of the certainty equivalents is $q \overline{\overline{R O I}}$, and the discount rate is $r_{f}$ which results from the use of certainty equivalents.

From Equation 5 it can be seen that the expected return on investment, the market price of risk, and the covariance are only contained in $\overline{\overline{R O I}}=\overline{R O I}-\lambda \rho$. The expected return on investment is thus only accounted for in its risk-adjusted form. The value of equity naturally increases along with the risk-adjusted expected return on investment. Therefore, the lower the systematic risk in the return on investment, represented by $\rho$, the higher the value of equity.

The total value of the business at time $h$, which is denoted by $V_{h}$, is then calculated by adding the values of equity and debt, each at time $h$ :

$$
V_{h}=V_{E Q, h}+D_{h}
$$

### 5.3 Cost of Equity

By means of the equity value from Equation 5 one can derive the cost of equity for each period. Following standard literature (Miles and Ezzel 1980) the cost of equity at time $h, k_{E, h}$, is the (conditional) expected return for equity holders over the period from $h$ to $h+1$ :

$$
k_{E, h}=\frac{E_{h}\left[F C F_{h+1}^{E}+V_{E Q, h+1}\right]}{V_{E Q, h}}-1
$$

As demonstrated
in

Appendix 2 we can show that:
$k_{E, h}=q \overline{R O I}+\left(r_{f}-q \overline{\overline{R O I}}\right) \overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}$
Equation 6
Equation 6 shows that the cost of equity is independent of time and state. It is the sum of the expected growth rate plus a non-negative adjustment which is driven by the structure of the free cash flows to equity owners. As can be seen, the cost of equity is a function of the risk-free interest rate $r_{f}$, the market price of risk $\lambda$, the covariance $\rho$, the expected return on investment $\overline{R O I}$-the previous three variables being included in $\overline{\overline{R O I}}$-the capital structure $v$, the plowback ratio $q$, the equity interest rate $r_{e}$ and the two tax rates. We point out that as we used the forward rates $k_{E}(s, t)$ to derive the equity value $V_{E Q, h}$, and $V_{E Q, h}$ to derive the cost of equity $k_{E, h}$. This cost of capital implicitly results from and includes the forward rates $k_{E}(s, t)$.

### 5.4 Market-to-Book Ratio

In addition to the cost of equity, an endogenous market-to-book ratio $M B R_{h}$ arises from Equation 5:

$$
M B R_{h}=\frac{V_{E Q, h}}{E Q_{h}}=\frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}
$$

As can be seen, the market-to-book ratio is independent of time and state. Let us compare this market-to-book ratio with the well-known market-to-book ratio from the Gordon constant growth dividend discount model (which is also independent of time and state) (Gordon 1962):

$$
M B R=\frac{\overline{R O E}-\bar{g}}{k_{E}-\bar{g}}
$$

Where $\overline{R O E}$ is the expected return on book equity, $\bar{g}$ is the expected growth rate, and $k_{E}$ is the cost of equity capital.

Using $\overline{R O E}, \bar{g}$, and $k_{E}$ consistent with our model, we will see that the market-to-book ratio from Equation 7 is equivalent to the market-to-book ratio from the Gordon constant growth dividend discount model. The return on book equity in each year $h$ is the ratio of earnings after taxes in year $h, E A T_{h}$, and equity at the beginning of year $h, E Q_{h-1}$ :

$$
R O E_{h}=\frac{E A T_{h}}{E Q_{h-1}}=R O I_{h}(1+v)\left(1-\tau_{K}\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)
$$

To obtain the expected return on equity one has to replace $R O I_{h}$ by $\overline{R O I}$ :

$$
\overline{R O E}=\overline{\operatorname{ROI}}(1+v)\left(1-\tau_{K}\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)
$$

The expected growth rate is $\bar{g}=q \overline{R O I}$ (see Section 0 ) and the cost of equity can be seen from Equation 6. Plugging in these values in the Gordon market-to-book ratio formula gives:

$$
\begin{aligned}
M B R=\frac{\overline{R O E}-\bar{g}}{k_{E}-\bar{g}} & =\frac{\overline{R O I}(1+v)\left(1-\tau_{K}\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)-q \overline{R O I}}{\left(r_{f}-q \overline{\overline{R O I}}\right) \overline{\operatorname{ROI}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)} \overline{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)}}= \\
& =\frac{\overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)}}{r_{f}-q \overline{\overline{R O I}}}
\end{aligned}
$$

Which is equal to the market-to-book-ratio in Equation 7.
Note that our market-to-book-ratio in Equation 7 also corresponds to a riskadjusted representation of the market-to-book ratio from the Gordon dividend discount model: by a slight modification of our market-to-book ratio in Equation 7 we receive:

$$
\begin{aligned}
M B R & =\frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}= \\
& =\frac{\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)-q \overline{\overline{R O I}}}{r_{f}-q \overline{\overline{R O I}}}=\frac{\overline{\overline{R O E}}-\overline{\bar{g}}}{r_{f}-\bar{g}}
\end{aligned}
$$

Where $\overline{\overline{R O E}}$ represents the risk-adjusted expected return on equity (which is obtained by replacing $R O I_{h}$ by $\overline{\overline{R O I}}$ in the equation for the return on book equity) and $\overline{\bar{g}}=q \overline{\overline{R O I}}$ stands for the risk-adjusted expected growth rate (see Section 0). The riskadjusted representation requires the use of a risk-adjusted expected return on equity $\overline{\overline{R O E}}$ instead of $\overline{R O E}$, as well as discounting with the risk-free rate instead of $k_{E}$. As in the dividend discount model, the (risk-adjusted) expected growth rate has to be subtracted in both numerator and denominator.

To sum up, this section covered the valuation of a firm in a tax system with imputed interest on equity using the equity method. The next two sections will deal with alternative DCF valuation methods, namely the Adjusted Present Value (APV) method and the entity method. By means of the APV method we are able to distil the contribution of equity tax shields to the total business value. Under the entity method we adjust the weighted average cost of capital, which is frequently used in industry, to a tax system with imputed interest on equity.

## Section 6: APV Method

In its basic form, the APV method is based on Myers (1974). The source of this approach lies in a model world with only one business tax with deductible debt
interest. However, the APV approach has been applied also to real-world and far more complex tax systems. ${ }^{9}$

The objective of this section is to extend the APV approach to a tax regime with imputed interest on the stock of equity. The tax benefit arising from equity financing has to be added to the existing tax benefit arising from debt financing. Due to the regulations governing imputed interest on equity, an adjustment to account for this imputed interest is also necessary in the case of unlevered companies. Owing to the fact that even businesses financed solely with equity are not homogenous (it includes businesses with different equity levels as well as different equity growth rates and thus different equity tax shields), a fictitious unlevered company that does not claim equity tax shields has to be selected as a common point of reference. Therefore, if the APV approach is applied to a tax regime with imputed interest on equity, the following equation must be used:

$$
V_{A P V, h}=V_{h}(U)+V_{h}(T S D)+V_{h}(T S E)
$$

Where:
$V_{A P V, h}=$ Market value (as of time $h$ ) of the levered company.
$V_{h}(U)=$ Market value (as of time $h$ ) of a company which is unlevered but otherwise equivalent and does not claim the tax benefit from imputed interest on equity.
$V_{h}(T S D)=$ Value at time $h$ of the tax shield arising from debt.
$V_{h}(T S E)=$ Value at time $h$ of the tax shield arising from imputed interest on equity.

The goal of this section is the valuation of the individual components.

[^5]
### 6.1 Valuation of the Unlevered Company that does not Claim Equity Tax Shields

In contrast to the levered company, the unlevered (but otherwise equivalent) company at each time $h$ has equity $E Q_{h}^{U}$, which is equal to the total capital of the levered business:

$$
E Q_{h}^{U}=T C_{h}=E Q_{h}+D_{h}=E Q_{h}(1+v)
$$

For the purpose of valuation, it is necessary to examine the free cash flow of the unlevered company for any year $t$, denoted by $F C F_{t}^{U}$. For the same reasons as in Section 5, the free cash flow is the difference between earnings after taxes and equity increase. The EBIT of the unlevered company in year $t$, which is by definition equal to EBT in the same year, is:

$$
E B I T_{t}^{U}=E Q_{t-1}^{U} R O I_{t} .
$$

Once corporate tax is subtracted, the earnings after taxes of this unlevered company are $E A T_{t}^{U}=E Q_{t-1}^{U} R O I_{t}\left(1-\tau_{K}\right)$. In order to obtain the free cash flow $F C F_{t}^{U}$, the equity increase, $q E B I T_{t}^{U}=q E Q_{t-1}^{U} R O I_{t}$, has to be subtracted:

$$
F C F_{t}^{U}=E Q_{t-1}^{U} R O I_{t}\left(1-\tau_{K}\right)-q E Q_{t-1}^{U} R O I_{t}=E Q_{t-1}^{U} R O I_{t}\left(1-\tau_{K}-q\right)
$$

Equation 8
Recurring to Equation 1 to Equation 3 yields: ${ }^{10}$

$$
\begin{aligned}
& a=1-\tau_{K}-q \\
& b=0
\end{aligned}
$$

By substituting for $a$ and $b$ in Equation 2 we get:

$$
k_{U}(s, t)=\left(1+r_{f}\right) \frac{\left(1-\tau_{K}-q\right) \overline{R O I}}{\left(1-\tau_{K}-q\right) \overline{\overline{R O I}}}-1=\left(1+r_{f}\right) \frac{\overline{\overline{R O I}}}{\overline{\overline{R O I}}}-1 \quad \text { for } s=t
$$

[^6]$$
k_{U}(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}-1 \quad \text { for } s<t
$$

Where $k_{U}(s, t)$ is the risk-adjusted discount rate (for the period from $s-1$ to $s$ ) for valuating the free cash flow of the unlevered company that occurs in year $t$.

The reason for the difference in $k_{U}(s, t)$ between the cases $s=t$ and $s<t$ again lies in the fact that risk is resolved differently. It can be shown that, given $\rho>0$, $k_{U}(s, t)$ is smaller in the years $s<t$ than for $s=t$. Thus, less risk is resolved in each year $s<t$ than in the final year $s=t$.

When $s<t$ the discount rate $k_{U}(s, t)$ is identical to the one under the equity method, $k_{E}(s, t)$. This is because for $s<t$ the systematic risk in the free cash flow of the unlevered company is identical to that in the free cash flow of the levered business under the equity method, since the returns on investment enter into the free cash flow indirectly via the level of equity for both companies. For $s=t k_{U}(s, t)$ is different from $k_{E}(s, t)$ due to a different systematic risk, with the ratio $\overline{R O I} / \overline{\overline{R O I}}$ reflecting the relation of the actual expected return on investment to the risk-adjusted expected return on investment.

The connection between the discount rates $k_{U}(s, t)$ and $k_{E}(s, t)$ for $s<\mathrm{t}$ and $s=t$ can be explained in greater detail as follows: the difference between the levered and the unlevered company consists on the one hand in the stock of equity [i.e., $E Q_{h}^{U}=$ $\left.E Q_{h}(1+v)\right]$, while on the other hand the unlevered company is subject to different tax treatment. This arises from the lack of debt tax shields as well as our definition of the unlevered company under the APV method (equity tax shields are not claimed). The difference in equity levels is irrelevant because it has no influence on how much systematic risk is resolved in the individual years (the equity level drops in the
derivation of the discount rate). The difference in tax treatment (elimination of tax shields for the unlevered company) only comes into play in the final year ( $s=t$ ).

Substituting for $a$ and $b$ in Equation 3, then gives the value of the unlevered company at time $h$ :

$$
V_{h}(U)=\frac{E Q_{h}^{U} \overline{\overline{R O I}}\left(1-\tau_{K}-q\right)}{r_{f}-q \overline{\overline{R O I}}}=\frac{E Q_{h}(1+v) \overline{\overline{R O I}}\left(1-\tau_{K}-q\right)}{r_{f}-q \overline{\overline{R O I}}}
$$

The economic interpretation corresponds to its counterpart under the equity method.

### 6.2 Valuation of Debt Tax Shields

The debt tax shields in year $t$ are $T S D_{t}=D_{t-1} r_{f} \tau_{K}=E Q_{t-1} v_{f} \tau_{K}$ so that we can use Equation 1 to Equation 3 with $a=0$ and $b=v_{f} \tau_{K}$. Substituting in Equation 2 yields:

$$
\begin{array}{ll}
k_{T S D}(s, t)=\left(1+r_{f}\right) \frac{r_{f} \tau_{K}}{v r_{f} \tau_{K}}-1=r_{f} & \text { for } s=t \\
k_{T S D}(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}-1 & \text { for } s<t
\end{array}
$$

Where $k_{T S D}(s, t)$ is the risk-adjusted discount rate (for the period from $s$ - 1 to $s$ ) for valuating the debt tax shields that occur in year $t$.

Thus, for $s<t$ the discount rate $k_{T S D}(s, t)$ is identical to its counterpart under the equity method, $k_{E}(s, t)$. In the case $s=t$ the discount rate is the risk-free interest rate $r_{f}$. Economically speaking, for $s<t$ debt is default risk-free, but the stock of debt is outstanding and therefore the resulting tax benefit from debt is not because due to the assumption of a constant capital structure the volume of debt fluctuates with the volume of equity capital. Owing to the constant capital structure, the systematic risk of debt tax shields is equivalent to that of equity under the equity method. Using the discount rate for the levered company is (for $s<t$ ) identical to using the discount rate
for the unlevered company. Therefore, this result is consistent with the results of Miles and Ezzell (1980), Harris and Pringle (1985), Ruback (1995), and Richter (2002).

For $s=t$, debt interest at the end of year $t$ is the product of debt at the beginning of year $t$ and the riskless rate. Furthermore, as debt is default risk-free, debt interest and hence also the tax benefits from debt interest are predictable by one year. In the final year $t$ the systematic risk in debt tax shields is thus 0 , which explains use of the riskless rate as a discount rate in the final year.

Again substituting for $a$ and $b$ in Equation 3 yields the value of all debt tax shields as of time $h$ :

$$
V_{h}(T S D)=\frac{E Q_{h} v r_{f} \tau_{K}}{r_{f}-q \overline{\overline{R O I}}}
$$

The numerator contains the debt tax shields for the first year after time $h$. A riskadjustment is not necessary for these tax shields because according to our model (see Section 2), these can be used in any case and $E Q_{h}$ and therefore $D_{h}$ is known at time $h$, thus making the tax shields certain in the first year after $h$. Due to the assumption of a constant capital structure, the tax shields in the ensuing years are subject to the same risk as equity is (with a one year time lag). For this reason, the growth rate to be used is once again the risk-adjusted growth rate $q \overline{\overline{R O I}}$. Finally, it is easy to see that for $q=0$, which implies constant debt $V_{h}(T S D)$, equals $D_{h} \tau_{K}$ corresponding to the standard case dealt with in literature (Modigliani and Miller 1963).

### 6.3 Valuation of Equity Tax Shields

The valuation of the equity tax shields is also performed with the methodology described above. If all tax effects (including the reduced tax on the imputed interest on the stock of equity) are taken into account, the net tax benefit of equity in year $t$ is:

$$
T S E_{t}=E Q_{t-1} r_{e}\left(\tau_{K}-\tau_{S}\right)
$$

Thus, we can use Equation 1 to Equation 3 with:

$$
\begin{aligned}
& a=0 \\
& b=r_{e}\left(\tau_{K}-\tau_{S}\right)
\end{aligned}
$$

Therefore, the discount rate for equity tax shields, $k_{T S E}(s, t)$, is:

$$
\begin{array}{ll}
k_{T S E}(s, t)=\left(1+r_{f}\right) \frac{r_{e}\left(\tau_{K}-\tau_{S}\right)}{r_{e}\left(\tau_{K}-\tau_{S}\right)}-1=r_{f} & \text { for } s=t \\
k_{T S E}(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}-1 & \text { for } s<t
\end{array}
$$

As can be seen, the discount rates for equity and debt tax shields are the same for all years $(s<t, s=t)$. Furthermore, for $s<t k_{T S E}(s, t)$ equals its counterpart under the equity method, $k_{E}(s, t)$ and the discount rate for the unlevered company, $k_{U}(s, t)$.

The reason for the equality of $k_{T S E}(s, t)$ and $k_{T S D}(s, t)$ is the equivalence of systematic risk of equity tax shields and debt tax shields. A constant capital structure at book values exposes equity and debt tax shields to the same source of variation, namely the evolution of the stochastic return on investment of the business under consideration. In the last period $s=t$ e.g., in both cases the stock of equity/debt is already known, leaving no more uncertainty to be resolved.

By plugging in $a$ and $b$ into Equation 3, the value of the equity tax shields as of time $h$ is:

$$
V_{h}(T S E)=\frac{E Q_{h} r_{e}\left(\tau_{K}-\tau_{S}\right)}{r_{f}-q \overline{\overline{R O I}}} .
$$

Again, the derivation is based on a growing perpetuity. The numerator contains the equity tax shields from the first year after time $h$. As for the debt tax shields in the first year after time $h$, no risk adjustment is necessary because the equity interest in
this year is computed from the equity stock at time $h$. Equity tax shields grow at the risk-adjusted expected growth rate $q \overline{\overline{R O I}}$ in the ensuing years, as is the case for the free cash flow under the equity method.

### 6.4 APV Method - Summary

Summing up, the value of the business using the APV method is the total value of the unlevered business that does not claim equity tax shields and the value of debt and equity tax shields:

$$
V_{A P V, h}=V_{h}(U)+V_{h}(T S D)+V_{h}(T S E)
$$

Proposition 1: The value of the business under the APV method, $V_{A P V, h}$, equals the value of the business under the equity method, $V_{h}$. The proof of Proposition 1 can be found in Appendix 3. Proposition 1 is consistent with Chambers et al., (1982) or Lewellen and Emery (1986).

## Section 7: Entity Method

As for the third DCF method, the entity method, the objective is to determine the weighted average cost of capital (WACC) in consideration of imputed interest on the stock of equity. Furthermore, we want to verify that the value $V_{\text {entity }, h}$ resulting from discounting expected unlevered free cash flows, $E\left[F C F_{t}^{U}\right]$, with the WACC corresponds to $V_{h}=V_{E Q, h}+D_{h}$.

In order to determine the WACC, we again follow the common methodology of deducing the weighted average cost of capital (see Miles and Ezzell 1980, equations 21 and 22) by computing the weighted average cost of capital at time $h, w_{h}$, as expected return for equity and debt holders together over the period from $h$ to $h+1$ :

$$
w_{h}=\frac{E_{h}\left[F C F_{h+1}^{U}+\left(V_{E Q, h+1}+D_{h+1}\right)\right]}{\left(V_{E Q, h}+D_{h}\right)}-1
$$

The right-hand term corresponds to the required rate of return at time $h$ of a levered company deducting imputed interest on equity: the numerator corresponds to the expected cash flow to equity and debt holders at time $h+1$ and the denominator stands for the value of the company at time $h$. This implies that all effects induced by financing decisions (i.e., debt and equity tax shields) are exclusively included in the discount rate, $w_{h}$.

Using for $V_{E Q, h}$ the value derived in Section 0 and some algebraic rearrangements provide us with the weighted average cost of capital for a company with infinite life:

$$
w_{h}=q \overline{R O I}+\left(r_{f}-q \overline{\overline{R O I}}\right) \frac{\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)}{\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+r_{f} \tau_{K}}
$$

Equation 9
The complete derivation is shown in Appendix 4.
As can be seen from Equation 9, the weighted average cost of capital is time and state independent. As with the cost of equity, the weighted average cost of capital is the sum of the expected growth rate plus a non-negative adjustment that depends on the structure of the cash flows to debt and equity holders (see Equation 8). The WACC is a function of the risk-free interest rate $r_{f}$, the market price of risk $\lambda$, the covariance $\rho$, the expected return on investment $\overline{R O I}(\lambda, \rho$, and $\overline{R O I}$ being included in $\overline{\overline{R O I}}$ ), the capital structure $v$, the plowback ratio $q$, the equity interest rate $r_{e}$, and the two tax rates.

Based on this representation of the WACC, it can be shown that $w_{h}$ can also be represented as a weighted average of the cost of equity and debt. Using the cost of equity, derived in Section 0, the WACC can be restated as follows (see Appendix 5):

$$
w=(1-n)\left[k_{E}-\frac{r_{e}\left(\tau_{K}-\tau_{S}\right)}{M B R}\right]+n\left[r_{f}\left(1-\tau_{K}\right)\right]
$$

Where $n$ stands for the market value debt ratio. Thus, the WACC is a market value weighted average of the cost of equity and the cost of debt, both adjusted for the values of their respective tax shields. The adjustment for the debt tax shields $-r_{f} \tau_{K}$ is well-known from the textbook formula. The adjustment for the equity tax shields depends on the equity interest rate applied to the stock of equity and the tax differential. The division by $M B R$ is required because equity interest is based on book equity instead of equity at market values.

By introducing and fully specifying the appropriate adjustment for the cost of equity in a world with imputed interest on the stock of equity, we extend the existing literature on cost of capital relations and thereby generalise the well-known textbook formula introduced by Modigliani and Miller (1963).

Finally, we can use the WACC to obtain the value of the firm under the entity method:

$$
V_{\text {entity }, h}=\sum_{t=h+1}^{\infty} \frac{E_{h}\left[F C F_{t}^{U}\right]}{(1+w)^{t-h}}
$$

Proposition 2: The value of the business under the entity method, $V_{\text {entity,h }}$, equals the value of the business under the equity method, $V_{h}$. Proposition 2 is consistent with the existing literature (Chambers et al., 1982; or Lewellen and Emery, 1986). A proof is provided in Appendix 6.

## Conclusion

This paper integrates a tax regime with imputed interest on the stock of equity into discounted cash flow business valuation. The equity method, the APV method, and the entity method are presented with attention to imputed equity interest. On the basis of Fama (1977), the discount rates for the valuation of free cash flows and of tax benefits arising from debt and equity financing are derived from the intertemporal evolution of the covariance between the book return on investment and the market return. A closed-form solution is derived for the value of equity and that of equity tax shields. Furthermore, we prove that a market-to-book ratio consistent with the one resulting from the Gordon constant growth dividend discount model follows from our model. Moreover, we show that the discount rates appropriate for the equity tax shields equal those for the debt tax shields. We also derive the WACC in a tax regime with imputed interest on the stock of equity and present the adjustment to the cost of equity that is necessary to allow the computation of the WACC as a weighted average of the cost of equity and cost of debt. Finally, we want to point out that the methodology used in this paper, including the cost of capital formulae (cost of equity capital and WACC), can be equally used for capital budgeting purposes.

## Appendix 1: Fama Technology—Derivation of Discount Rates

The goal of this appendix is to derive the discount rates for a cash flow using the Fama valuation model. From Fama's equation (29) in connection with equation (30), both on page 13 in Fama (1977), the forward rates can be derived for a cash flow $Z_{t}$ occurring at time $t$ :

$$
\frac{1-\lambda \operatorname{cov}\left[\varepsilon_{s, t} r_{M, s}\right\rfloor}{1+r_{f}}=\frac{1}{1+k(s, t)} \text {, so that } k(s, t)=\frac{1+r_{f}}{1-\lambda \operatorname{cov}\left[\varepsilon_{s, t}, r_{M, s}\right]}-1
$$

Equation 10
Where $r_{M, s}$ is the return of the market portfolio in period $s$ and $\varepsilon_{s, t}$ is an expectations adjustment variable (with zero mean) that measures the incremental information (change in expectation) in period $s$ on the cash flow $Z_{t}$ :

$$
\varepsilon_{s, t}=\frac{E_{s}\left[Z_{t}\right]}{E_{s-1}\left[Z_{t}\right]}-1
$$

In the derivation of the discount rates for a cash flow stream of the structure $Z_{t}=E Q_{t-1}\left(a R O I_{t}+b\right)$, the two cases $s=t$ and $s<t$ have to be distinguished.

For all years $s<t$ :

$$
\varepsilon_{s, t}=\frac{E_{s}\left[E Q_{t-1}\left(a R O I_{t}+b\right)\right]}{E_{s-1}\left[E Q_{t-1}\left(a R O I_{t}+b\right)\right]}-1
$$

Because $E Q_{t-1}$ and $R O I_{t}$ are uncorrelated,

$$
\begin{aligned}
\varepsilon_{s, t} & =\frac{E_{s}\left[E Q_{t-1}\right]\left(a E_{s}\left[R O I_{t}\right]+b\right)}{E_{s-1}\left[E Q_{t-1}\right]\left(a E_{s-1}\left[R O I_{t}\right]+b\right)}-1= \\
& =\frac{E Q_{s-1}\left(1+q R O I_{s}\right)(1+q \overline{R O I})^{t-s-1}(a \overline{R O I}+b)}{E Q_{s-1}(1+q \overline{R O I})^{t-s}(a \overline{R O I}+b)}-1=\frac{1+q R O I_{s}}{1+q \overline{R O I}}-1
\end{aligned}
$$

The covariance between $\varepsilon_{s, t}$ and the market return in the same year, $r_{M, s}$, is:

$$
\operatorname{cov}\left[\varepsilon_{s, t}, r_{M, s}\right]=\frac{q \operatorname{cov}\left[R O I_{s}, r_{M, s}\right]}{1+q \overline{R O I}}=\frac{q \rho}{1+q \overline{R O I}}
$$

So that from Equation 10:

$$
k(s, t)=\frac{1+r_{f}}{1-\lambda \frac{q \rho}{1+q \overline{R O I}}}-1=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{R O I}-q \lambda \rho}-1
$$

Now defining $\overline{\overline{R O I}}=\overline{R O I}-\lambda \rho$ yields:

$$
k(s, t)=\left(1+r_{f}\right) \frac{1+q \overline{R O I}}{1+q \overline{\overline{R O I}}}-1
$$

For the case $s=t$ we get:

$$
\begin{gathered}
\varepsilon_{s, t}=\frac{E Q_{t-1}\left(a R O I_{t}+b\right)}{E_{t-1}\left[E Q_{t-1}\left(a R O I_{t}+b\right)\right]}-1=\frac{E Q_{t-1}\left(a R O I_{t}+b\right)}{E Q_{t-1}(a \overline{R O I}+b)}-1=\frac{a R O I_{t}+b}{a \overline{R O I}+b}-1 \\
\operatorname{cov}\left[\varepsilon_{s, t}, r_{M, s}\right]=\frac{a \operatorname{cov}\left[R O I_{t}, r_{M, t}\right]}{a \overline{R O I}+b}=\frac{a \rho}{a \overline{R O I}+b}
\end{gathered}
$$

So that from Equation 10:

$$
\begin{aligned}
k(s, t) & =\frac{1+r_{f}}{1-\lambda \frac{a \rho}{a \overline{R O I}+b}}-1=\left(1+r_{f}\right) \frac{a \overline{R O I}+b}{a \overline{R O I}+b-\lambda a \rho}-1=\left(1+r_{f}\right) \frac{a \overline{R O I}+b}{a(\overline{R O I}-\lambda \rho)+b}-1= \\
& =\left(1+r_{f}\right) \frac{a \overline{R O I}+b}{a \overline{\overline{R O I}}+b}-1
\end{aligned}
$$

## Appendix 2: Derivation - Cost of Equity

The goal of this appendix is to derive the cost of equity $k_{E, h}$ :

$$
k_{E, h}=\frac{E_{h}\left[F C F_{h+1}^{E}+V_{E Q, h+1}\right]}{V_{E Q, h}}-1=\frac{E_{h}\left[F C F_{h+1}^{E}\right]}{V_{E Q, h}}+\frac{E_{h}\left[V_{E Q, h+1}\right]}{V_{E Q, h}}-1
$$

Substituting for $F C F_{h+1}^{E}$ using Equation 4 and substituting Equation 5 for $V_{E Q, h}$ gives after computing the conditional expectation:

$$
\begin{aligned}
& k_{E, h}=\frac{E Q_{h} \overline{\left.\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)\right]}}{E Q_{h} \frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}+} \\
& +\frac{E Q_{h}(1+q \overline{R O I}) \frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}}{E Q_{h} \frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}}-1 \\
& \\
& =q \overline{\overline{R O I}}+\frac{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)}{\overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-r_{f}\left(1-\tau_{K}\right)}} \\
& r_{f}-q \overline{\overline{R O I}} \\
&
\end{aligned}
$$

## Appendix 3: Proof - Proposition 1

## (Equivalence of Value from Equity Method and APV Method)

In order to demonstrate that the sum of the three components under the APV method corresponds to the value derived with the equity method, we use the fact that the value of the business under the equity method is equal to the value of equity plus the value of debt, $V_{h}=V_{E Q, h}+D_{h}$. As $V_{E Q, \mathrm{~h}}$ is given by Equation 5 and $D_{h}=E Q_{h} v$.

$$
\begin{aligned}
V_{h} & =\frac{\left.E Q_{h} \overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]}{r_{f}-q \overline{\overline{R O I}}}+E Q_{h} v \\
& =\frac{E Q_{h}\left[\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]+E Q_{h} v\left(r_{f}-q \overline{\overline{R O I}}\right)}{r_{f}-q \overline{\overline{R O I}}}
\end{aligned}
$$

The three components from the APV method are as follows: The value of the unlevered business is according to Section 0 :

$$
V_{h}(U)=\frac{E Q_{h}(1+v) \overline{\overline{R O I}}\left(1-\tau_{K}-q\right)}{r_{f}-q \overline{\overline{R O I}}}
$$

The value of the debt tax shields is from Section 0:

$$
V_{h}(T S D)=\frac{E Q_{h} v r_{f} \tau_{K}}{r_{f}-q \overline{\overline{R O I}}}
$$

The value of the equity tax shields is (see Section 0):

$$
V_{h}(T S E)=\frac{E Q_{h} r_{e}\left(\tau_{K}-\tau_{S}\right)}{r_{f}-q \overline{\overline{R O I}}}
$$

When these three components are added and then equated to the value of the business under the equity method, $\frac{E Q_{h}}{r_{f}-q \overline{\overline{R O I}}}$ cancels out, resulting in:

$$
\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)+v\left(r_{f}-q \overline{\overline{R O I}}\right)=
$$

$=\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+v r_{f} \tau_{K}+r_{e}\left(\tau_{K}-\tau_{S}\right)$

Because all components which contain either the risk-free interest rate $r_{f}$ or the equity interest rate $r_{e}$ cancel out, the result is:

$$
\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]-v q \overline{\overline{R O I}}=\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)
$$

Factoring out $\overline{\overline{R O I}}$ gives:

$$
\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)=\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)
$$

Thus the APV method and the equity method deliver the same results.
Q.E.D.

## Appendix 4: Derivation of WACC

The weighted average cost of capital at time $h, w_{h}$, according to Miles/Ezzell (1980) is the expected return for equity and debt holders together over the period from $h$ to $h+1$ :

$$
w_{h}=\frac{E_{h}\left[F C F_{h+1}^{U}+\left(V_{E Q, h+1}+D_{h+1}\right)\right]}{\left(V_{E Q, h}+D_{h}\right)}-1
$$

As $V_{E Q, h}=E Q_{h} M B R$ and $D_{h}=E Q_{h} v$, we can express the market value of the business $\left(V_{E Q, h}+D_{h}\right)$ as $E Q_{h}(M B R+v)$. This gives:

$$
w_{h}=\frac{E_{h}\left[F C F_{h+1}^{U}+E Q_{h+1}(M B R+v)\right]}{E Q_{h}(M B R+v)}-1
$$

Since expected book equity grows at a rate of $q \overline{R O I}$ (see Section 3), we obtain:

$$
w_{h}=\frac{E_{h}\left[F C F_{h+1}^{U}\right]}{E Q_{h}(M B R+v)}+\frac{E Q_{h}(1+q \overline{R O I})(M B R+v)}{E Q_{h}(M B R+v)}-1=\frac{E_{h}\left[F C F_{h+1}^{U}\right]}{E Q_{h}(M B R+v)}+q \overline{R O I}
$$

Plugging in the respective expressions for the expected free cash flow of the unlevered company using Equation 8 and computing the expectation yields

$$
w_{h}=q \overline{R O I}+\frac{E Q_{h}^{U} \overline{R O I}\left(1-\tau_{K}-q\right)}{E Q_{h}(M B R+v)}=q \overline{R O I}+\frac{(1+v) \overline{R O I}\left(1-\tau_{K}-q\right)}{M B R+v}
$$

Because $E Q_{h}^{U}=E Q_{h}(1+v)$. As furthermore it follows from Equation 7 that:

$$
\begin{aligned}
& M B R+v=\frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}+v= \\
& =\frac{\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+r_{f} \tau_{K}}{r_{f}-q \overline{\overline{R O I}}}
\end{aligned}
$$

The weighted average cost of capital is:

$$
w_{h}=q \overline{R O I}+\left(r_{f}-q \overline{\overline{R O I}}\right) \frac{\overline{\operatorname{ROI}}(1+v)\left(1-\tau_{K}-q\right)}{\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{s}\right)+r_{f} \tau_{K}}
$$

## Appendix 5: Proof - Representation of WACC as a Weighted Average

The purpose of this appendix is to show that the WACC can be represented as:

$$
w=(1-n)\left[k_{E}-\frac{r_{e}\left(\tau_{K}-\tau_{S}\right)}{M B R}\right]+n\left[r_{f}\left(1-\tau_{K}\right)\right]
$$

Where $n$ stands for the debt ratio at market values and 1-n corresponds to the equity ratio at market values. As equity at market values at each time $t$ is $E Q_{t} M B R$ and debt at (both book and) market values at each time $t$ is $E Q_{t} v$, the market value weight of equity is represented by the expression $M B R /(M B R+v)$ and the market value weight of debt is $v /(M B R+v)$. So the above equation translates into:

$$
w=\frac{M B R}{M B R+v}\left[k_{E}-\frac{r_{e}\left(\tau_{K}-\tau_{S}\right)}{M B R}\right]+\frac{v}{M B R+v}\left[r_{f}\left(1-\tau_{K}\right)\right]
$$

By rearranging terms:

$$
w=\frac{1}{M B R+v}\left[k_{E} M B R-r_{e}\left(\tau_{K}-\tau_{S}\right)+v_{f}\left(1-\tau_{K}\right)\right]
$$

In a first step we substitute Equation 6 for $k_{E}$ :

$$
w=\frac{1}{M B R+v}\left[\begin{array}{l}
\left.q \overline{R O I} M B R+\left(r_{f}-q \overline{\overline{R O I}}\right) \overline{\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v_{f}\left(1-\tau_{K}\right)} M B R\right] \\
-r_{e}\left(\tau_{K}-\tau_{S}\right)+v_{f}\left(1-\tau_{K}\right)
\end{array}\right]
$$

As from Equation 7 we know that:

$$
\frac{\overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)}{r_{f}-q \overline{\overline{R O I}}}=M B R
$$

This is equivalent to:

$$
w=\frac{1}{M B R+v}\left[\begin{array}{l}
q \overline{R O I} M B R+\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{s}\right)-v r_{f}\left(1-\tau_{K}\right) \\
-r_{e}\left(\tau_{K}-\tau_{S}\right)+v r_{f}\left(1-\tau_{K}\right)
\end{array}\right]
$$

As both terms $r_{e}\left(\tau_{K}-\tau_{S}\right)$ cancel out and the same is true for the terms $v_{f}\left(1-\tau_{K}\right)$, this equals:

$$
w=\frac{q \overline{R O I} M B R+\overline{R O I}\left[(1+v)\left(1-\tau_{K}\right)-q\right]}{M B R+v}
$$

Since $-q \overline{R O I}=-q(1+v) \overline{R O I}+q v \overline{R O I}$, this is equivalent to:

$$
w=\frac{q \overline{R O I} M B R+q v \overline{R O I}+\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)}{M B R+v}=q \overline{R O I}+\frac{\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)}{M B R+v}
$$

From Appendix 4 we know that:

$$
M B R+v=\frac{\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+v_{f} \tau_{K}}{r_{f}-q \overline{\overline{R O I}}}
$$

Now we can substitute for $M B R+v$ in the equation above:

$$
\begin{aligned}
w & =q \overline{R O I}+\frac{\overline{\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)}}{\left.\frac{r_{f}-q \overline{R O I}}{\overline{R O}} 1+v\right)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+v_{f} \tau_{K}} \\
& =q \overline{\overline{R O I}}+\left(r_{f}-q \overline{\overline{R O I}}\right) \frac{\overline{\operatorname{ROI}}(1+v)\left(1-\tau_{K}-q\right)}{\overline{\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+r_{f} \tau_{K}}}
\end{aligned}
$$

Which equals the WACC derived in Equation 9.
Q.E.D.

## Appendix 6: Proof - Proposition 2

(Equivalence of Value from Entity Method and Value from Equity Method)
The purpose of this appendix is to show that $V_{\text {entity, } h}=V_{h}=V_{E Q, h}+D_{h}$. The value of the firm under the entity method is:

$$
V_{\text {entity }, h}=\sum_{t=h+1}^{\infty} \frac{E_{h}\left[F C F_{t}^{U}\right]}{(1+w)^{t-h}}
$$

From Equation 8 we know that the free cash flows of the unlevered business are $F C F_{t}^{U}=E Q_{t-1}^{U} R O I_{t}\left(1-\tau_{K}-q\right)$. As $E Q_{t}^{U}=E Q_{t}(1+v)$, the growth rate of $E Q_{t}^{U}$ equals that of $E Q_{t}$. Thus, the expected free cash flows of the unlevered business (conditional on time $h$ ) are a growing perpetuity with $E_{h}\left[F C F_{h+1}^{U}\right]=E Q_{h}(1+v) \overline{R O I}\left(1-\tau_{K}-q\right)$ and a growth rate of $q \overline{R O I}$.

Therefore:

$$
V_{\text {entity }, h}=\frac{E Q_{h}(1+v) \overline{R O I}\left(1-\tau_{K}-q\right)}{w-q \overline{R O I}}
$$

Substituting for $w$ as derived in Equation 9 yields:

$$
V_{\text {eniti }, \mathrm{h}}=\frac{E Q_{h}(1+v) \overline{R O I}\left(1-\tau_{K}-q\right)\left[\overline{\operatorname{ROI}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+v_{f} \tau_{K}\right]}{\overline{\operatorname{ROI}}(1+v)\left(1-\tau_{K}-q\right)\left(r_{f}-q \overline{\overline{R O I}}\right)}
$$

As $\overline{R O I}(1+v)\left(1-\tau_{K}-q\right)$ cancels out, this is equivalent to

$$
V_{\text {enitiv, }, h}=\frac{E Q_{h}\left\lfloor\overline{\overline{R O I}}(1+v)\left(1-\tau_{K}-q\right)+r_{e}\left(\tau_{K}-\tau_{S}\right)+r_{f} \tau_{K}\right]}{r_{f}-q \overline{\overline{R O I}}}
$$

Since:

$$
-E Q_{h} \overline{\overline{R O I}}(1+v) q=-E Q_{h} \overline{\overline{R O I}} q-E Q_{h} v q \overline{\overline{R O I}}
$$

and

$$
E Q_{h} v_{f} \tau_{K}=-E Q_{h} \nu r_{f}\left(1-\tau_{K}\right)+E Q_{h} \nu r_{f}
$$

This is equivalent to:

$$
\begin{aligned}
V_{\text {entit }, h} & =\frac{\left.\left.E Q_{h} \overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]\right]+E Q_{h} v\left(r_{f}-q \overline{\overline{R O I}}\right)}{r_{f}-q \overline{\overline{R O I}}} \\
& =\frac{\left.E Q_{h} \overline{\overline{R O I}}\left[(1+v)\left(1-\tau_{K}\right)-q\right]+r_{e}\left(\tau_{K}-\tau_{S}\right)-v r_{f}\left(1-\tau_{K}\right)\right]}{r_{f}-q \overline{\overline{R O I}}}+E Q_{h} v
\end{aligned}
$$

From Equation 5 and the fact that $D_{h}=E Q_{h} v$ we see that:

$$
V_{\text {entity }, h}=V_{E Q, h}+D_{h}
$$

Q.E.D.

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[^0]:    ${ }^{1}$ This proposal has recently also found the backing of the German Finance minister Hans Eichel (see Der Standard of February 9, 2005).

[^1]:    ${ }^{2}$ For and overview see Brealey and Myers 2000, Ross et al. 2002.

[^2]:    ${ }^{3}$ As a generalization of this model, the assumption of a 2 - or multi-stage model would be possible. The model presented in this paper would correspond to the final stage of such a multi-stage model and is thus a prerequisite for the implementation of a multi-stage model. The methodology to be applied to the valuation of payments in the previous stages is identical to that in the final stage, described in the following.
    ${ }^{4}$ Papers in the line of Schwartz and Aronson (1967) find empirical evidence that firms exhibit a tendency to maintain a constant capital structure over time. As imputed equity interest is calculated on the basis of book equity, a constant capital structure at book values is assumed.
    ${ }^{5}$ This assumption is necessary to get the closed-form solutions in the following Sections. Serial correlation could be incorporated into the model, using the same methodology - however at the expense of analytical tractability.
    ${ }^{6}$ An alternative assumption would involve loss compensation. This manner of modelling is common in the valuation of (debt) tax shields (Modigliani and Miller 1963, Myers 1974, Miles and Ezzell 1980, Fernandez 2002, Richter 2002).

[^3]:    ${ }^{7}$ Alternative models can be found in Stapleton (1971), Bierman and Hass (1973), Bogue and Roll (1974), Stevens (1974), Bierman and Smidt (1975), Rubinstein (1976), Myers and Turnbull (1977), Breeden and Litzenberger (1978), and Breeden (1979). These models can be distinguished by the number of cash flows to be valued, by their (in-)consistency with the single-period Sharpe/Lintner/Mossin CAPM, by the (non-)stationarity of the CAPM parameters and in case of nonstationary parameters, by the question whether they are deterministic or stochastic. Furthermore, continuous and discrete-time models can be distinguished.

[^4]:    ${ }^{8}$ In analogy to existing literature (Damodaran 1992) we assume that there are no changes in accounting provisions in any year.

[^5]:    ${ }^{9}$ See Monkhouse (1997) and Drukarczyk and Richter (1995) for the Australian tax system, and Hachmeister (1996) for the German tax system.

[^6]:    ${ }^{10}$ As $E Q_{t}^{U}=E Q_{t}(1+v)$, the growth rate of $E Q_{t}^{U}$ equals that of $E Q_{t}$.

