Nominal versus Indexed Debt: A Quantitative Horse Race

Laura Alfaro[**]
Harvard Business School

Fabio Kanczuk[***]
Universidade de São Paulo

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Abstract

There are different arguments in favor and against nominal and indexed debt which broadly include the incentive to default through inflation versus hedging against unforeseen shocks. We model and calibrate these arguments to assess their quantitative importance. We use a dynamic equilibrium model with tax distortion, government outlays uncertainty, and contingent-debt service, which we take to mean nominal debt. In the model, the benefits of defaulting through inflation are tempered by higher future interest rates. We obtain that calibrated costs from inflation more than offset the benefits from hedging. We further discuss sustainability of nominal debt in developing (volatile) countries.

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** lalfaro@hbs.edu. Harvard Business School, Boston, MA 02163, Tel: 617-495-7981, Fax: 617-495-5985
*** kanczuk@usp.br. Dept of Economics, Universidade de Sao Paulo, Brazil

1 Introduction

In the 18th and 19th centuries, Britain issued indexed perpetuities to finance its deficit. In the 1970s, Britain experimented with inflation-linked debt and since 1982 has offered inflation-linked gilts.[1] On January 29, 1997, U.S. Treasurer Lawrence Summers auctioned $7 billion of 10 year Treasury Inflation-Protected Securities (TIPS). Before that, U.S. debt was virtually all non-indexed nominal debt. Brazil, in contrast, after introducing inflation-indexed securities in 1964, is now struggling to reduce the level of indexed-debt. In 2003, less than 10% of the Brazilian debt was nominal (not indexed). What explains these differences? What are the criteria for choosing the optimal amount of nominal debt? In what sense do these factors depend on the characteristics of the country?

There are different arguments for or against nominal and indexed debt, which broadly include the incentive to default through inflation versus the advantage of hedging against unforeseen shocks.[2] In this paper, we run a quantitative horse race between these different rationales. We do this by modeling the different arguments and calibrating the model in order to assess the quantitative importance of each. Despite the vast theoretical literature, few studies have analyzed the trade-offs between nominal and indexed debt by constructing calibrated economy laboratories, a methodology that yields a sharper understanding of the quantitative forces involved in economic processes.

Economists have long proposed using indexed bonds and hence have long been puzzled that most governments in the world do not index government debt.[3] The most compelling argument in favor of indexation is that it eliminates a government's incentive to inflate the economy in order to reduce the real cost of nominal liabilities. Lucas and Stokey (1983), for example, show that the government has an incentive to inflate nominal liabilities unless prices are predetermined or all

[1] See Barro (1997).
[2] Lucas and Stokey (1983), Bohn (1988, 1990), Calvo and Guidotti (1990, 1993), Barro (1979, 1997), among others, have studied aspects of debt management decisions.
[3] Proponents of indexed bonds include strong voices such as Jevons, Marshall, Keynes, Fisher, Musgrave, Friedman, and Tobin. See Fischer (1983) for an overview of the main arguments in favor and against indexed debt.

distortionary taxation can be avoided. Since neither assumption seems realistic, indexed government debt should be strictly preferred to nominal debt.[4]

Nominal debt, however, allows the government the possibility of hedging against unexpected shocks that affect the fiscal budget and hence reduce tax distortions. Bohn (1988), for example, argues that nominal debt provides a valuable insurance against the budgetary effects of economic fluctuations. Nominal debt may be a desirable form of funding because of the covariance of inflation with government spending: high government spending tends to go with high inflation. Since nominal bonds pay poorly in real terms when inflation is surprisingly high, nominal debt has some of the characteristics of government contingent debt: nominal bonds allow the government to partially default via inflation. Similarly, Calvo and Guidotti (1993) show that optimal monetary policy calls for the inflation tax to be employed to pay part of the unanticipated fiscal deficit in a given period. Of course, the incentive to inflate remains. As Calvo and Guidotti (1993) mention: "enlarging the nominal base to reduce inflation and tax volatility may tempt the devil. Thus, the benefits from issuing nominal bonds need to be carefully weighted against the costs resulting from time-inconsistency."

Summing up, defaulting on nominal liabilities can generate a welfare loss that must be quantitatively weighed against the gains from tax smoothing. This is the task we undertake in this paper. A good starting point to approach this problem is the model developed by Grossman and Van Huyck (1988). These authors treat sovereign debt as a contingent claim.[5] This assumption follows from the observation of two of the main stylized facts behind sovereign borrowing and defaulting. These are (i) defaults are usually partial and associated with identifiable bad states of nature; (ii) sovereign states often are able to borrow soon after default. Hence, in their model, sovereign defaults occur as outcomes of debt-servicing obligations that are implicitly contingent on the realized state of the world. In their view, lenders sharply differentiate excusable defaults, which are justifiable when

[4] See also Calvo (1978, 1988) and Missale and Blanchard (1994).
[5] As Eaton et al. (1986, p.482) note, "though indeed the borrower is required to service a debt, there is no way that, in general, the borrower can be forced to do so under all contingencies. Debt and equity are both contingent claims, although they clearly differ in the nature of the contingencies involved."

associated with implicitly understood contingencies, from debt repudiation, which would be inexcusable.[6]

This paper quantitatively applies this model to the problem of determining the optimal amount of contingent debt, which we take to mean nominal debt. For this purpose, we construct and calibrate a dynamic equilibrium model that focuses on the role of contingent debt to intertemporally reduce tax distortions. In the model, nominal debt allows accommodating negative shocks to the government budget. However, the model also recognizes that contingent debt is associated with incentive problems. In fact, the basic intuition behind the model is that when the method of financing is too convenient, the government is likely to abuse it – an intuition that has already been pointed out by such memorable economists as Adam Smith and David Ricardo.[7]

In order to capture this trade-off, we propose an adverse-selection-like information structure, similar to the one used by Cole, Dow and English (1995) and Alfaro and Kanczuk (2005a). We assume there are two types of sovereigns. "Bad" sovereigns are extremely impatient and always default, independently of the state of the economy; "good" sovereigns default optimally (excusably) in order to smooth tax distortions. The equilibrium interest rate is determined by lenders, who signal-extract the type of government[8] After a default occurs, lenders become more uncertain about the type of the sovereign, and tend to charge higher interest rates. Consequently, the benefits of defaulting are tempered by higher interest rates in the future which worsen the government's financing problem. Noteworthy, this feature of the model is consistent with another stylized fact of sovereign borrowing, namely, that countries with histories of defaults are charged higher interest rates than countries with no repayment difficulties.

We then calibrate our model to match the U.S. economy. Our objective is to find the optimal contingency structure, i.e the optimal amounts of indexed and non-indexed debt. We find that for a

[6] As a consequence, defaults are equilibrium phenomena, which is consistent with the stylized facts.
[7] "Were the expense of war to be defrayed always by revenue raised within the year…wars would in general be more speedily concluded and less wantonly undertaken" (Adam Smith, 1791).

wide range of parameters, *in the absence of inflation costs*, the only equilibrium of the model is one in which the sovereign defaults in all states. That is, nominal debt is not sustainable unless inflation is costly. This finding is not a reincarnation of the Bulow and Rogoff (1989) result regarding debt sustainability. As Grossman and Han (1999) show, a very appealing property of a contingent service model of sovereign debt, like ours, is that it can support positive amounts of debt even if the sovereign can save after defaulting.[9] Hence, the result about the importance of inflation costs for nominal debt sustainability is a quantitative result specific to our calibration.

More interestingly, when inflation costs are calibrated according to the empirical evidence, our artificial economy experiments suggest that the optimal amount of nominal debt is zero. That is, for any amount of nominal debt, the inflation costs more than offset the benefits from tax smoothing. Our quantitative results, consequently, are at odds with the empirical evidence since, as mentioned, nominal debt tends to be the rule rather than the exception in most countries.[10]

We also study a situation in which the government *must* use some amount of non-indexed debt. In this case, the optimal structure is to have no indexed debt at all. This perhaps surprising result has a simple explanation. Given certain government expenditure volatility, the required inflation rate in order to smooth taxes in bad periods is smaller the greater the nominal debt. Therefore, the cost associated with inflation, which is quantitatively the most important one, is smaller the greater the non-indexed debt proportion.

Finally, we calibrate our artificial economy to match an emerging market. We chose the Brazilian case for various reasons. The country introduced indexation relatively early (1964) and, as

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[8] As experience illustrates, lenders face considerable uncertainty about the government's characteristics and preferences.

[9] There are two main assumptions in the literature that studies sovereign debt as way to smooth taxes or consumption. In contingent debt models, such as Eaton and Gersovitz (1981) and Chari and Kehoe (1993), smoothing is achieved by making debt issuance contingent on the realization of income: the sovereign issues (retires) new debt whenever it has a low (high) realization of income. In contingent service models, such as Grossman and Van Huyck (1988), smoothing is achieved by making debt-servicing contingent on the realization of income: the sovereign services its debt in full only when hit by a high realization of income; in the event of a low realization of income, the sovereign defaults either partially or fully.

[10] We do not model additional costs associated with the use of indexed debt, such as those related to the availability of reliable price indices. See Fischer (1983) for further discussion.

mentioned before, is attempting to reduce the levels of indexed debt. Due to the high inflation experience, public debt in Brazil is virtually all linked to short-term interest rates (overnight).[11] To match the Brazilian economy stylized facts, we calibrate the model with government outlays ten times more volatile than in the United States. For this economy, we obtain that the interest on nominal debt required to mitigate the moral hazard incentive to inflate (which we label agency cost) is so high that it is sometimes impossible to raise enough taxes in order to service the debt. More specifically, there is a boundary on the maximum amount of sustainable nominal debt for a given degree of agency costs. Thus, given the existing volatility of the government outlays, the attempt of issuing contingent debt in Brazil, as sometimes proposed, may not be feasible.

In terms of the framework, our work is similar to Chari and Kehoe's (1993) analysis of debt sustainability, where taxes on labor are the only other way to finance government outlays in addition to defaulting on the debt. The model shares with Alfaro and Kanczuk (2005a) the computational strategy and definition of equilibrium. Our result about the superiority of indexed debt due to the quantitative importance of incentive problems corroborate the argument outlined in Barro (1997). In addition, our work relates to the broader debate around the flexibility of monetary policy. Calvo and Guidotti (1993) argue that the time-inconsistency problems temper, but do not necessarily invalidate the case for flexible monetary policy and that monetary policy should be endowed with some flexibility. In contrast, our quantitative results caution against excess flexibility.

The rest of the paper is organized as follows. The model is developed in section 2. Section 3 presents the computational implementation and equilibria. Section 4 defines the data and calibration. The results are discussed in section 5. Section 6 concludes.

[11] In Brazil, indexing to short-term interest rates was preferred over indexing to consumer price indexes because the latter only captures inflation with one-month delay. For more on Brazilian debt, see Giavazzi and Missale (2004).

2 Model

Our economy is populated by a government (sovereign) who borrows funds and taxes labor income from a *continuum* of identical infinitely lived consumers (private sector), in order to finance an exogenous path of public expenditures. The sovereign type evolves over time. He can be one of two types, "good" or "bad," which describe conditions that reflect his impatience. Whereas "bad" sovereigns are extremely impatient and choose to default at any time independently of the state of the economy, "good" sovereigns may or may not choose optimally to default on their commitments.

In each period t, the private sector supplies labor and capital to produce a single output good, according to a Cobb-Douglas production function. The output can be used for private consumption, private investment, or government consumption.

Let c_t, h_t, i_t, g_t and k_t denote the per capita levels of consumption, labor, investment, government spending and capital, and let A_t denote a technology parameter. Feasibility requires,

$$c_t + i_t + g_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{1}$$

The preferences of each consumer are given by

$$U = E\sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{2}$$

with,

$$u(c,h) = c - \varphi \frac{h^{1+1/\xi}}{(1+1/\xi)} \tag{3}$$

where φ, $\xi > 0$, and $\beta \in (0, 1)$. As discussed below, the assumption that consumption enters linearly in the period utility function has the advantage of simplifying the determination of the equilibrium interest rates without affecting the tax-smoothing motive of the model. The parameter ξ is referred to as the Frisch's intertemporal labor elasticity.

The per capita level of government consumption in each period, denoted by g_t, is exogenously specified. We assume g_t can take a finite number of values and evolve over time according to a Markov transition matrix with elements π (g_i , g_j). That is, the probability that $g_{t+1} = g_j$ given that $g_t =$

g_i is given by the matrix π element of row i and column j. Government consumption is financed through a proportional tax on labor income and with debt. Let τ_t denote the tax on labor income in period t. Government debt represents claims to consumption units in the next period. These claims can be either contingent or non-contingent. Denote the amount of contingent debt and noncontingent debt purchased by consumers respectively by b_{t+1} and d_{t+1}. Let $(1 + r_t)$ and $(1 + \rho_t)$ denote the (inverse of the) prices of these claims and v_t denote the marginal product of capital. Let $\theta_t \in [0, 1]$ denote the default rate on contingent government debt outstanding in period t. The private sector's budget constraint is given by,

$$c_t + i_t + b_{t+1} + d_{t+1} = w_t(1-\tau_t)h_t + v_t k_t + (1-\theta_t)(1+r_t)b_t + (1+\rho)d_t \qquad (4)$$

Notice that r_t corresponds to the (contractual) interest rate on the contingent debt, ρ is the riskless interest rate, and θ_t is the default rate (one can also think of θ as a tax on debt). We assume θ_t can take only two values, $\theta_t \in \{0, \chi\}$, where $\chi \in [0, 1]$, which correspond respectively to the cases of not defaulting and defaulting.

As in Grossman and Van Huyck (1988), we consider a case in which the levels of capital and contingent and noncontingent debt are fixed and equal to k, b, and d respectively. This assumption implies that debt and capital cannot be used to smooth taxes. This assumption, which greatly reduces the calculation burden, is a necessary step to making equilibrium computable. As Grossman and Han (1999) point out, however, this assumption is much less restrictive than it seems. They show that, when government can save after defaulting, contingent debt (or capital) does not allow for any additional tax smoothing. In contrast, contingent service may engender more tax smoothing than the one already attained through savings.

Because the capital level is constant, investment level must be equal to $i = \delta k$. Hence, the private sector's choices are to decide how much to work and how much to charge for government bonds.

The sovereign's preferences are given by

$$U = E\sum_{t=0}^{\infty}\beta_{sov}^{t}u(C_t, H_t) \tag{5}$$

where $u(.,.)$ is as in the consumers' preferences, but C_t and H_t denote the aggregate per capita levels of consumption and labor, respectively. The sovereign budget constraint is,

$$b + d = g_t - \tau_t w_t H_t + (1-\theta_t)(1+r_t^*)b + (1+\rho)d \tag{6}$$

where we have already suppressed the time subscripts for debt levels.

There are two types of sovereigns who differ in the parameter β_{sov}. The "good" sovereign has the same discount factor of the private agents, $\beta_{sov} = \beta$. In contrast, the "bad" sovereign fully discounts the future: $\beta_{sov} = 0$. A direct consequence of this assumption is that the "bad" sovereign always defaults.[12] The assumption of different types of sovereigns is meant to capture the uncertainty lenders face about the sovereigns' preferences and willingness to honor debts. One can also consider that the same leader stays in power, but there is turnover among key advisors.

As in Cole, Dow and English (1995) and Alfaro and Kanczuk (2005a), the government type evolves according to a Markov process (of common knowledge) with the transition probabilities given by[13]

$$\begin{bmatrix} 1-\psi & \psi \\ \psi & 1-\psi \end{bmatrix} \tag{7}$$

That is, a "good" type at t remains a good type at $(t + 1)$ with probability $1 - \psi$, and transitions to a bad type with probability ψ. Similarly, a "bad" type at t remains a bad type at $(t + 1)$ with probability $(1 - \psi)$ and transitions to a "good" type with probability ψ. For this process to display persistence, we assume $\psi < 1/2$. We name ψ the *agency cost parameter*. Higher values of ψ imply

[12] This assumption captures the flavor of Grossman and Van Huyck's (1988) "excusable defaults." A bad type always defaults (even in good times), which is not excusable. The good type might default, but only in relatively bad times ("excusable default").

[13] Notice, however, that decisionmakers care about their consumption independent of whether they are in power or not. That is, they do not discount future consumption by the likelihood of their type changing, nor do they think their discount rate is time varying.

that the current government type carries less information about the future sovereign type. Thus, when making decision, consumers are more uncertain about the sovereign's type. In each period, the timing is as follows (see Figure 1). At the beginning of period t, the country inherits amounts of contingent and non-contingent debt equal to b and d, which bear an interest rate of r_t and ρ_t respectively. Then nature reveals the sovereign type and the public expenditure level. After observing the public expenditure level, the sovereign decides whether to default or not, θ_t, and, consequently, how much to tax labor, τ_t. Based on these decisions, consumers decide how much to work, h_t. They also update their information about the sovereign's type and decide how much to charge for the next period contingent debt, r_{t+1}.

The assumption regarding the utility functional form greatly simplifies the solution. The Euler equation for labor (labor supply) jointly with the usual firms' maximization problem (labor demand)[14] imply,

$$H_t = [(1-\tau_t)(1-\alpha)A_t k^{\alpha} / \varphi]^{1/(1/\xi+\alpha)} \tag{8}$$

Additionally, the Euler equation for consumption implies that the private sector behaves as risk-neutral lenders, with an opportunity cost given by ρ_t. As previously mentioned, we assume that the private sector cannot directly observe the government's type. Therefore, the lending rate r_t depends on the perceived likelihood of default.

We find it convenient to express lenders' information about the likelihood of default by defining two probabilities, p_t and q_t. Let $p_t \in [0,1]$ be the probability that the sovereign in period t, at the time of choosing whether or not to default, is of the "good" type. Let $q_t \in [0,1]$ be the probability that a sovereign will default at time t *given* that the sovereign is of the "good" type. The perceived probability of default at t is given by $1 - p_t(1 - q_t)$.

For lenders to be indifferent between the riskless asset and the contingent debt, it must be that $1+\rho = p_t(1-q_t)(1+r_t) + [1-p_t(1-q_t)](1+r_t)(1-\chi)$, which implies that the interest rate is given by

$$1 + r_t = (1+\rho)/[1-\chi(1-p_t(1-q_t))] \tag{9}$$

A final assumption concerns the technology parameter A_t. We assume:

$$A_t = \exp[-\lambda(r_t - \rho)] \tag{10}$$

This expression for productivity, a departure from standard modeling, is meant to capture the costs of inflation that have been documented in the literature. It should be seen as a shortcut that dispenses the need for explicitly considering money stocks in the analysis, greatly reducing the computation burden.[15] We do experiment with cases for which $\lambda = 0$, where this effect is absent. But, as we later show, positive values for λ seem to be an important factor for the qualitative nature of the equilibria.

To understand how the model works, consider a good sovereign that chooses to default. If a sovereign defaults, choosing $\theta_t = \chi$, expression (6) indicates that the country will enjoy a lower tax distortion today, τ_t. This decision might affect the future interest rate lenders charge and, thus, future taxes. Indeed, when lenders extract the information from the default in order to set the next period's interest rate, they will most likely consider the possibility that this period sovereign was of the "bad" type. This in turn, given (7), implies a higher probability that the sovereign country also will be of the "bad" type next period. Consequently, the private sector chooses to charge a higher interest rate (expression 9).

As a consequence of defaulting, there are lower tax distortions today in exchange for higher tax distortions in the future. Following the usual assumptions regarding preferences and technology, welfare is higher for smoother tax profiles. Thus, default is a more likely outcome when the state of the economy is such that, *for a constant θ,* the government expenditure today is higher than the expected government expenditure in the future.

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[14] As usual, we assume there is a continuum of firms (or a single price taker firm), and the demand for labor equates wages with the marginal product of labor: $w_t = A_t(1 - \alpha)(k/H)^{\alpha}$.

3 Computational Implementation and Equilibrium

The model described is a stochastic dynamic game. We restrict our attention to the Markov perfect equilibria, which we discuss next. We start by defining the *state of the economy* at period t as the ordered set (g_{t-1}, g_t, p_t), and the *excusable default set*, D, as

$$D = \{(g_{-1}\ g, p) \textit{ such that lenders believe that the good type will default}\}$$

where, for any period, g_{-1} denotes the technology in the previous period, and g and p denote, respectively, this period technology and lenders' assessment of the probability that the sovereign is of the good type. The excusable default set, part of the lender's strategy, corresponds to all states of the economy in which lenders believe that the sovereign will default. In Grossman and Van Huyck's (1988) language, D corresponds to the states of the economy in which defaults are excusable.

Given D, we can write the lenders' future probabilities assessments as a function of the state and of the sovereign's action as

$$p_{t+1} = 1 - \psi \text{ , if } \theta_t = 0, \tag{11a}$$

$$p_{t+1} = \psi \text{ , if } \theta_t = \chi \text{ and } (g_{t-1}, g_t, p_t) \notin D, \tag{11b}$$

$$p_{t+1} = p_t(1 - \psi) + (1 - p_t)\psi \text{ , otherwise} \tag{11c}$$

which corresponds to simple Bayesian updating, and

$$q_{t+1} = \sum_{(g_t, g_{t+1}, p_{t+1}) \in D} \pi(g_{t+1}, g_t) \tag{12}$$

which comes straight from the definition of the excusable default set.

Notice that the lender's strategy is completely determined by the set D and the expressions (9), (11), and (12). As a consequence, given D, we can write the sovereign's problem as

$$V(g_{t-1}, g_t, p_t) = Max_{\theta_t} \{u(C_t, H_t) + \beta_{sov} EV(g_t, g_{t+1}, p_{t+1})\} \tag{13}$$

such that (3), (6), (8), (9), (10), (11), and (12) hold.

Now we are ready for the definition of equilibrium.

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[15] In a typical cash-in-advance economy such as Cooley and Hansen (1989) and Lucas (2000), the consumer's

A *Markov perfect equilibrium* is an excusable default set D, a value function V, and a policy function θ such that, given D, θ *is* a solution for the problem (13), and

$$\theta\,(g_{-1}, g, p) = \chi \quad \text{, for all } (g_{-1}, g, p) \in D, \text{ and}$$

$$\theta\,(g_{-1}, g, p) = 0 \quad \text{, otherwise.}$$

Although in a slightly different format much in line with one of "recursive competitive equilibrium," this definition is not different from the usual Markov perfect equilibrium definition. Given the private sectors' strategy (the set D and the expressions for p, q, H and r) and the state, the sovereign maximizes utility. Given the sovereign strategy (θ) and the state, the private sector optimally chooses how much to work, and it is indifferent between buying contingent debt and earning the riskless rate. Hence, its strategy is also optimal.

4 Data and Calibration

We calibrate our model to match two economies: the United States as an example of a mature economy and Brazil as an example of an emerging market. We used data from 1957 to 2003 to calibrate both economies. We calibrated our model so that each period corresponds to one year.

To calibrate the government expenditure, we use data on the ratio of government expenditure to GDP. This choice reflects an attempt to capture the tax smoothing motive, that is, the variation of government expenditures relative to that of tax revenues. Notice that in our model there are no technology shocks. These shocks have a direct impact on GDP and tax revenues. Thus, we "internalize" tax revenue shocks into the government expenditure shocks, by studying the ratio g_t/y_t. We assume g can take one of two possible levels, g_H and g_L, and assume that the transition probability matrix that defines the Markov process is symmetric. To calibrate the necessary parameters, we first detrend the g_t/y_t series. We then calculate their mean, standard deviation, and autoregressive coefficients. For both the United States and Brazil, the mean is about g/y = 20%, and the

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income decreases with nominal interest rates.

autoregressive coefficient is about 0.97. In contrast, the standard deviation is very different in the two countries: government expenditure in Brazil ($\sigma_{g/y\text{-}BRAZIL} = 0.40$) is about ten times more volatile than in the United States ($\sigma_{g/y\text{-}U.S.} = 0.04$). To match these facts, we set

$$\pi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$$

for both countries. For the U.S., we set $g_H/y = 0.208$ and $g_L/y = 0.192$. And for Brazil, we set $g_H/y = 0.28$ and $g_L/y = 0.12$.

Following the Real Business Cycle literature, we can calibrate most of the technology and preferences parameters. The (the inverse of the) price of the noncontingent debt is given by $\rho = 0.05$. This implies a discount parameter of $\beta = 0.95$. The depreciation level is $\delta = 0.05$ and $\alpha = 0.33$. We also use the Frisch elasticity proposed by Domeij and Floden (2003), $\xi = 0.5$, and that the average hours of work is $H = 0.3$.

We make many experiments for different values of the contingent debt level, *b,* but we always set the total debt level constant. We impose this assumption of a constant total debt in order to make more explicit the role of the noncontingent debt, *d*, and make welfare comparisons. In the United States, $(b + d)/y = 60\%$. In Brazil, the total debt is $(b + d)/y = 70\%$.

The total amount of debt allows us to determine the amount of tax revenues required to equilibrate the government budget constraint for a given level of government consumption, *g*. From this follows that the mean value of the tax rate on labor is $\tau = 35.1\%$. With this value and the Euler equation for labor, we can calibrate $\varphi = 8.71$.

Because the transition probabilities of government type are unobservable, we experiment with many different agency costs parameters (ψ). Recall that the higher ψ is, the more severe the punishments for not honoring the nominal debt (i.e., defaulting) are. As we discuss later in this section, our calibration exercises reveal this punishment, however, to be too small to sustain positive amounts of nominal debt in the U.S. economy, unless there are additional costs of inflation.

For computational reasons, we restrict the probability assessment, p, to only three values, ψ, $(1 - \psi)$, and their average, which is equal to 1/2. These three values, nevertheless, allow for interesting updating dynamics. For example, the punishment for defaulting in a state in which a sovereign is expected to default (an "excusable default" in the language of Grossman and Van Huyck) is different from that for defaulting in a state in which the sovereign is not expected to default (a debt "repudiation").

We assume the parameter χ is set in order to minimize the standard deviation of the tax rate. Since tax rates are different for each state, the calibration of χ will also be different in each equilibrium. In a sense, by allowing χ to depend on the equilibrium, we are capturing that fact the government can choose the default level. However, note that we are assuming that χ is *not* state-dependent. Thus, as the definition of equilibrium makes clear, the only government choice is, in fact, to default or not.

Finally, for the parameter λ, which corresponds to the costs of defaulting, we initially try $\lambda = 0$. This corresponds to the case of a pure contingent bond, with no inflation or other costs directly related to the default. In this case, the costs of a default would exclusively be related to the fact that the private sector tends to charge higher interest rates in the future, which would worsen the government finances. However, as discussed next, positive values for λ are important even for the qualitative results. We then use Lucas' (2000) result that a 10% reduction in inflation implies a welfare gain that corresponds to 1% of consumption (or GDP). Accordingly, we calibrate $\lambda = 0.1$. We interpret the case of $\lambda = 0.1$ to mean that the contingent bonds are nominal bonds.[16] Hence, defaults are necessarily associated with the inflation tax regardless of any other effects. This assumption of nominal bonds is the interpretation we pursue in this paper.

Table 1 contains the full list of parameter calibrations.

[16] In Missale and Blanchard (1994), very short term maturity is analogous to indexed debt as they define effective maturity ('differently from the conventionally measured maturity') as the effect of a change in inflation

5 Results

5.1 The U.S. Economy

a. Number of Equilibria

Our model is set up so that for any parameter there is always an equilibrium that corresponds to the case in which the sovereign defaults for all possible states. To observe that, consider that the excusable default set D contains all possible states. Then, from expression (12), one sees that $q = 1$, and from expression (9), that $1 + r_t = (1+\rho)/(1-\tau)$, for all states. These expressions indicate that punishment is independent of what a sovereign does or, better, that investors are not drawing any information from a sovereign's actions. Consequently, the sovereign has no incentive not to default and chooses to default in any state. This strategy validates the equilibrium.

This trivial equilibrium of "defaulting in all states" does not, however, capture the flavor of contingent-debt. Hence, we focus only on the other equilibria. It turns out that for the observed nominal debt levels in the U.S. economy, if we assume no defaulting costs ($\lambda = 0$), these other equilibria do not exist. This is the case for a wide range of different parameters values combinations: different nominal debt levels (b = 5%, 10%, 15%, …), and agency costs (ψ = 10%, 1%, 0.1%, …). It is important to note that this result is *not* a reincarnation of Bulow and Rogoff's (1989) theorem. Grossman and Han (1999) proved there were positive amounts of sustainable debt in an environment such as ours.[17] Contingent debt servicing permits more consumption smoothing than would saving and dissaving alone and thus, the sovereign would resist the temptation to repudiate its debt.[18] Our result is

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on the value of the debt. See Alfaro and Kanczuk (2005b) for a quantitative analysis of the arguments in favor and against short-term debt.

[17] After a sovereign country defaults in a state that does not belong to D it can still hold debt. However, since the sovereign will be considered to be of the bad type, the debt service is not *contingent* anymore: it is the same in any state of nature. Hence, a sovereign that inexcusably defaults cannot buy insurance policies like those proposed in Bulow and Rogoff's (1989) construction.

[18] As Grossman and Han (1999) explain, with contingent debt issuance once the sovereign reaches any positive debt ceiling repudiation would not reduce the possibilities for future consumption smoothing, whereas with debt contingent-services, debt servicing repudiation always would reduce the possibilities for future consumption smoothing. Because there is a finite limit to accumulated savings and debt, the possibility of contingent saving or

a quantitative one—for our economy, the *calibrated* value of debt is not sustainable if we assume that λ is zero.

If instead we consider inflation costs and set $\lambda = 0.1$, there are many equilibria. Table 2 reports the number of equilibria for a variety of parameters. Recall that for our calibration, there are 2 x 2 x 3 = 12 states of the economy. Each equilibrium is defined by the existence or not of default in each state. Thus, for each economy, there are at most 2^{12} possible equilibria.

Notice that in general there are more equilibria for low values of nominal debt and for low values of the agency cost parameter. This is so because lower debt levels and lower agency costs imply higher punishments for a default which, in turn, increases the probability that an equilibrium is sustainable. With lower agency costs, punishment is higher, since consumers have less doubt about the type of a sovereign that defaulted. Lower debt levels also imply in higher punishment, as we see next.

b. Equilibrium Selection

In this model, as in any repeated game, one lacks a good way to choose among the many possible equilibria. By changing the debt composition, the economy may exhibit different equilibria. These different equilibria display different default patterns, corresponding to different contingent claims and, in this sense, are not really comparable.

For our model, however, it turns out that there is only one equilibrium that appears in all the economies we studied (i.e., for all combinations of b and ψ). Thus, this equilibrium becomes the natural choice to make welfare comparisons. The following welfare comparisons, then, refer to this particular equilibrium.

Interestingly, the equilibrium common to all economies is a very simple one. It corresponds to a claim that is contingent only in the current state of government expenditure level. It determines that

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contingent debt issuance would not permit complete smoothing of consumption. Thus even if the sovereign could save and dissave, it would still be valuable to issue debt with contingent servicing in order to achieve more consumption smoothing.

there should be default if $g_t = g_H$, and that there should not be a default if $g_t = g_L$. This is so regardless of last period government expenditure and of any belief about the type of the sovereign.

To better characterize this equilibrium, tables 3 and 4 depict, respectively, the default rates and nominal interest rates for the case $\psi = 0.1\%$ and $b/y = 60\%$. These two tables have 12 cells, which correspond to all states of the economy, the combinations of possible values for g_{-1}, g and p. Notice that, as we just mentioned, the default rate is different from zero (i.e., there is default) whenever $g = g_H$, irrespectively of g_{-1} and p. In contrast, the equilibrium interest rate is different for all combinations of g_{-1} and p. Since the government expenditure process has some persistency, the previous period level contains information for this period level. That is, high government expenditure in $t - 1$ increases the probability that government expenditure in t is also high, and thus increases the default probability and the equilibrium interest rates. The prior information about the government being of the good type (p) also contains information about default, due to the government's type persistence. Hence, the higher p is, the higher the probability that government will be of the good type in the next period and, thus, the lower is the probability of default.

c. Welfare

To numerically calculate the welfare, we first obtain the invariant distribution of the occurrence of the possible 12 states and average their pay-offs, always considering that the government is of the good type. We then report welfare in consumption terms, as in Lucas (1987). We assume that the case in which there is only indexed debt (no nominal debt) corresponds to a consumption level equal to 100%. Then we report the welfare variation for all other parameter changes.

Table 5 depicts the welfare comparison for the different combinations of agency costs and debt composition. Notice first that all cases with nominal debt display a lower welfare level in relation

to the case with only indexed debt (no nominal debt). But also, welfare is higher when the nominal debt is higher.

The fact that lower agency costs imply higher welfare is not surprising. As agents can better identify the type of government, they would tend to charge lower interest rates when the sovereign is of the good type. With lower interest rates, welfare is higher both because government debt financing becomes easier (and thus tax rates become lower) and because the costs directly associated with inflation (the output drop) become smaller.

The result about the debt composition, in contrast, is a novel result derived from our experiments. There is no way to know *a priori* if more indexed debt implies in higher or lower welfare. To start with, it is not clear how the composition of debt impacts debt service and if it improves or worsens the public debt financing and the consequent tax distortions.

We believe the best way to grasp this result is by observing the default rates associated with each equilibrium. Table 6 depicts these default rates. Observe that there is a direct correspondence between welfare and default rates, i.e., the higher the default rates the lower the welfare level. We know that higher default rates imply higher equilibrium interest rates and, consequently, higher inflation costs. Hence, it seems that the inflation costs (the output drop) dominate all the other effects.

In terms of debt composition, one observes that the greater the stock of nominal debt, the lower the default rates. This is so because greater stock of nominal debt implies that the "inflation tax base" is bigger, that is, the same default rate implies in higher government revenues for greater nominal debt stocks. Thus, with greater nominal debt stock outstanding, the sovereign can smooth taxes with a lower inflation rate.

d. Optimal Debt Management

The above discussion assumed that the stock of nominal debt was constrained to be at most the stock of total outstanding debt. However, there is in principle no natural bound for the nominal

debt *b,* even if total debt, ($b + d$), is constant. For example, one could have a negative value for d, indicating that the sovereign is a debt creditor in indexed bonds. Interestingly, as noted above, the greater b the larger will be the inflation tax base. And a larger tax base implies that the same degree of labor tax smoothing could be obtained by smaller changes in inflation. In the limit, as b goes to infinity, the default rate goes to zero and perfect smoothing is still possible. Noteworthy, this result is opposite to a result in Persson, Persson and Svensson (1988), where the government, in order to reduce the incentive to generate surprise inflation, should buy nominal assets (become a net creditor in nominal bonds) and issue indexed debt.

Figure 2 depicts welfare for different debt structures. It shows that the as the amount of nominal debt increases, the welfare becomes close to the case with only indexed debt. For nominal debt values inferior to 60% of GDP, it is clear that the economy is better off having exclusively indexed debt. As we increase the amount of nominal debt up to 435% of GDP, welfare gradually improves. Notice that since we assumed a fixed amount of total debt equal to 60% of GDP, this case corresponds to one in which the government is the creditor of indexed bonds amounting to 375% of GDP. Even in this extreme case, the welfare level never surpasses that achieved when there is only indexed debt.

For nominal debt levels higher than 435% of GDP, the economy did not display this equilibrium anymore. This could be because the debt level is not sustainable (punishment costs are too small), or because of pure computational problems (numerical approximations). In either case, this huge amount of debt is not empirically relevant. Hence, as a conclusion, our quantitative results suggest that the best debt composition is issuing only indexed (non-contingent) debt.

As another experiment, consider that the United States increases the proportion of indexed debt. Suppose that although this policy is gradual, consumers do not anticipate it, such that our model is still an adequate tool to analyze welfare.[19] In the United States, the amount of indexed debt nominal

[19] If consumers did anticipate changes in government debt composition, their maximization problem would be potentially very different from the one we analyze.

debt in 2002 is close to 1.5% of GDP. Thus, the potential welfare increase of a change in the current composition of debt to one in which there is only indexed debt is 0.15% of the consumption level. However, if the transition process from nominal to indexed debt is gradual, the economy has to go through periods of low amounts of nominal debt before reaching the state in which all debt is indexed, as seen in Figure 2. This process implies considerable welfare losses in amounts close to 2% of the consumption level.

Thus, our model suggests very different recommendations, depending on whether the debt structures changes are swift or gradual. An abrupt switch from nominal to indexed debt could be beneficial. However, in a more realistic scenario of a gradual transition, this process could imply important welfare losses, and it would be safer to increase even more the fraction of nominal debt.

5.2 Debt sustainability in an emerging market: the case of Brazil

We now investigate the model implications for an emerging market. Our main concern in studying debt in an emerging economy is debt sustainability. The distinctive characteristic of most emerging markets economies is that volatility is much bigger than in the United States. In the case of Brazil, as noted before, the volatility of government expenditure (as a fraction of GDP) is about ten times bigger than in the United States.[20] Also as noted before, the Brazilian government stated its intention to increase the proportion of non indexed debt, but seems to have been unsuccessful in doing so. Our interpretation of this is that the Brazilian government is facing sustainability problem, which makes issuing nominal debt a difficult task. From the point of view of tax smoothing, the volatility of government expenditures need not imply a sustainability problem. If the government expenditures are more volatile, then the costs from not smoothing taxes should be bigger, and thus debt should be more easily sustainable. Thus, the sustainability problem comes from a different source, namely the inability of the government to raise enough revenue in order to service the debt. The intuition for this goes as

[20] For the calibration of the Brazilian economy, we follow Kanczuk (2004).

following. Higher volatility of government consumption implies that in order to smooth taxes it is necessary to use higher default rates. As a consequence, the equilibrium interest rates are also higher, and so are the debt services (in periods when there is no default). Thus, the labor tax rate that is needed to raise taxes enough to pay for the debt services is also higher. However, as usual, there is a limit in the tax revenue from labor taxes, which is given by a Laffer curve maximum point. If the labor tax revenues necessary to roll debt are higher than its maximum feasible amount, consumers will know that government will not be able to roll the debt. As a consequence, they expect the government to always default on debt, and the nominal debt ceases to be contingent on the state of nature.

In order to numerically analyze this effect, we compute the default rate consistent with the maximum feasible value for labor tax revenue. We denote this level as the "maximum feasible default rate." We compare this value against the equilibrium default rate that minimizes the labor tax variance in each equilibrium. If a particular equilibrium default rate is larger than the maximum feasible default rate then, we can conclude, this equilibrium is not really feasible.

Figure 3 shows these amounts for the case $\psi = 0.1\%$, which corresponds to an economy with (almost) no agency costs, and debt is more easily sustainable. For very low levels of debt it is possible to sustain some equilibrium. When the debt level is higher than a threshold, the minimum equilibrium default becomes higher than the maximum feasible default, and hence the equilibrium ceases to be feasible. In fact, sustainable equilibria cease to exist for higher nominal debt levels as the figure shows. That is, the amount of taxes required to service the debt become unfeasibly high.

More generally, one can determine the set of parameters for which some amount of nominal debt is sustainable. Figure 4 depicts this set. For low agency costs values, the equilibrium interest rates required to service the debt are lower. This softens up the government budget constraint and as a consequence, a greater amount of nominal debt becomes sustainable.

Turning our attention back to Brazil, as we mentioned before, the government has declared its intention to increase the amount of non-indexed debt, which was only about 10% of GDP, whereas total debt is 70% (see Ministerio da Fazenda (2004)). Our analysis suggests that this policy is simply

not feasible. The private sector would not be willing to buy bigger amounts of nominal debt at any interest rate, believing the government would never honor it. Hence, our results for volatile economies, with history of inflation and defaulting suggest that countries in difficulties issue indexed bonds simply because they cannot avoid it.[21]

6 Conclusions

In this paper, we used the methodology of simulating a calibrated artificial economy to study sovereign debt default. In particular, we ran a quantitative horse race between the main arguments in favor of and against nominal debt which are the incentive to default through inflation versus hedging against unforeseen shocks. We use a dynamic equilibrium model with tax distortion, government outlays uncertainty, and contingent debt service, which we take to mean nominal debt. In the model, the benefits of defaulting through inflation are tempered by higher future interest rates. We use the model to calibrate the case of the U.S. economy. We further discuss sustainability of nominal debt in volatile (developing) countries where issues related to debt sustainability have taken center stage among policy circles.

A noteworthy result of our calibration exercises is that with realistic parameter values, we obtain that equilibria with mostly nominal debt (like the one observed in the U.S.) cannot be sustained in Brazil. Our results not only cast doubt about the possibility of volatile developing countries, such as Brazil, of successfully reducing their indexed debt levels but also caution against the optimality of doing so from a welfare point of view. Indeed, our results suggest that state contingent contracts are not easy to sustain where incentive problems are present**.** In a broader sense, our results caution against accommodative debt policies. However, one should keep in mind that our results depend on many simplifications needed to keep the computational analysis manageable. We assumed that the debt levels were constant, we did not explicitly model the distortionary effects of inflation taxes, and

[21] Fischer (1983) wrote "Governments in inflationary difficulties issue indexed bonds and those that can avoid,

we completely abstracted from analyzing different debt maturity structures. Exploring these directions further is an important topic for future research.

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Table 1: Calibration

Preferences
b = 0.95 (time discount factor)
x = 0.5 (labor intertemporal elasticity)
j = 8.71 (labor disutility parameter)
Technology
° = 0.33 (capital share)
° = .05 (depreciation rate)
° = 0.1 (inflation cost on output)
U.S. Government
gH/y = 0.208 (high government expenditure level)
gL/y =0 .192 (low government expenditure level)
p = 0.95 (government expenditure persistency)
y = 0.1 (agency costs) (type persistency)
$(b + d)/y$ = 60% (total debt)
Brazilian Government
gH/y = 0.28 (high government expenditure level)
gL/y = 0.12 (low government expenditure level)
$(b + d)/y$ = 70% (total debt)

Table 2: Number of Equlibria for U.S. Economy

Nominal	Agency Costs (%)		
Debt (%)	0.1	1.0	10.0
10	5	5	5
20	5	5	1
30	5	4	2
40	3	3	2
50	3	3	2
60	3	3	2

Table 3: Default Rate (%) for U.S. Economy, *b* = 60%, ψ = .1%

		$g = g_L$	$g = g_H$
$p = \psi$	$g_{-1} = g_L$	0.00	1.70
	$g_{-1} = g_H$	0.00	1.70
$p = 1/2$	$g_{-1} = g_L$	0.00	1.70
	$g_{-1} = g_H$	0.00	1.70
$p = 1 - \psi$	$g_{-1} = g_L$	0.00	1.70
	$g_{-1} = g_H$	0.00	1.70

Table 4: Interest Rate (%) for U.S. Economy, b = 60%, ψ = .1%

		$g = g_L$	$g = g_H$
$p = \psi$	$g_{-1} = g_L$	6.82	6.82
	$g_{-1} = g_H$	6.81	6.81
$p = 1/2$	$g_{-1} = g_L$	6.77	6.77
	$g_{-1} = g_H$	5.95	5.95
$p = 1 - \psi$	$g_{-1} = g_L$	6.72	6.72
	$g_{-1} = g_H$	5.09	5.09

Table 5: Welfare Loss (% consumption) for U.S. Economy

Nominal	Agency Costs (%)		
Debt (%)	0.1	1.0	10.0
10	-0.77	-0.78	-0.91
20	-0.40	-0.41	-0.48
30	-0.27	-0.28	-0.33
40	-0.21	-0.21	-0.25
50	-0.17	-0.17	-0.21
60	-0.14	-0.15	-0.17

Table 6: Default Rate (%) for U.S. Economy

Nominal	Agency Costs (%)		
Debt (%)	0.1	1.0	10.0
10	8.9	8.9	9.5
20	4.7	4.8	5.1
30	3.2	3.3	3.5
40	2.5	2.5	2.6
50	2.2	2.2	2.1
60	1.7	1.7	1.8

Figure 1: Time Line

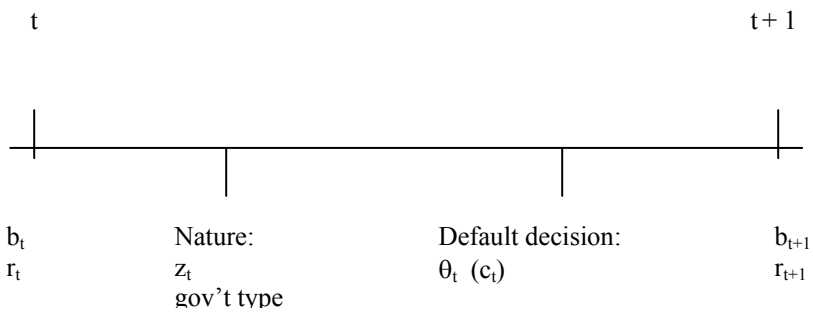

Figure 2: Welfare (% consumption) for U.S. Economy with ψ = 0.1%

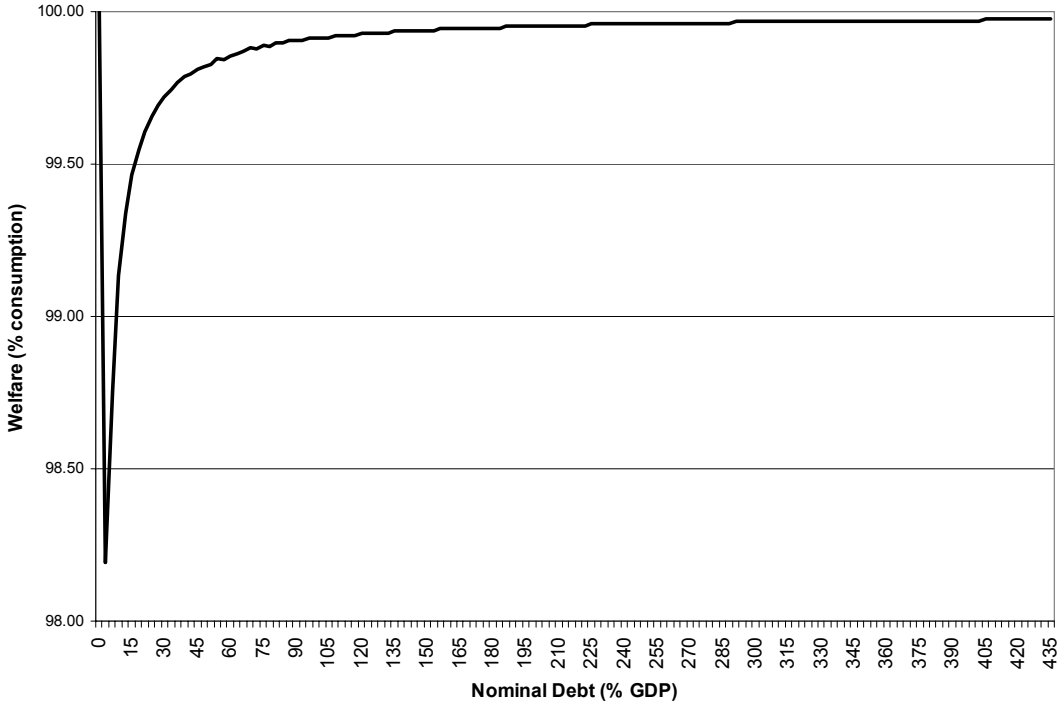

Figure 3: Brazilian Economy with Agency Cost (ψ) = 0.1%

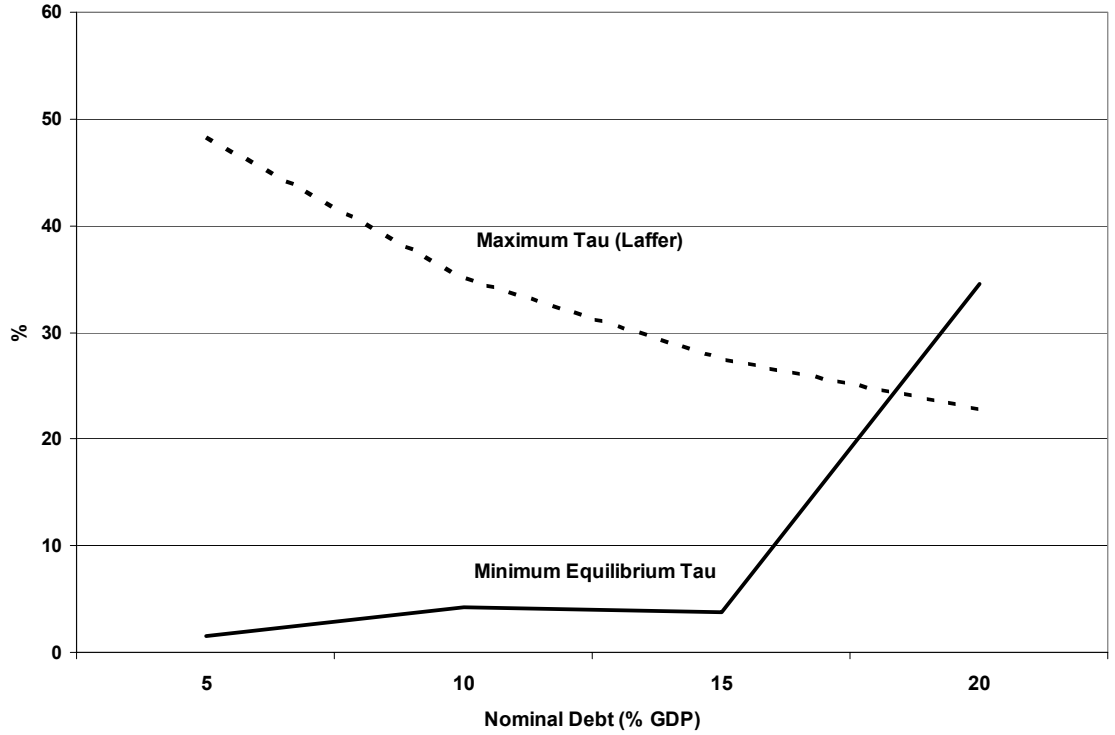

Figure 4: Brazilian Economy